Trigonometric Substitution

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- For integrals that involve an expression of the form $\sqrt{a^2 - x^2}$, try the substitution $x = a\sin\theta$, $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ then $dx = a\cos(\theta)d\theta$ and $\sqrt{a^2 - x^2} = a\cos(\theta)$.
- For integrals that involve an expression of the form $\sqrt{a^2 + x^2}$, try the substitution $x = a\tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ then $dx = a\sec^2(\theta)d\theta$ and $\sqrt{a^2 + x^2} = a\sec(\theta)$.
- For integrals that involve an expression of the form $\sqrt{x^2 - a^2}$, try the substitution $x = a\sec\theta$, $0 \leq \theta < \frac{\pi}{2}$ or $\pi \leq \theta < \frac{3\pi}{2}$then $dx = a\sec(\theta)\tan(\theta)d\theta$ and $\sqrt{x^2 - a^2} = a\tan(\theta)$.

Example 1: Substitution $x = a\sin\theta$

Find $\displaystyle\int \frac{4}{x^2\sqrt{16 - x^2}}dx.$

Solution

Write the solution here

Example 2: Substitution $x = a\tan\theta$

Find $\displaystyle\int \frac{x^3}{4\sqrt{4+x^2}}dx.$

Solution

Write the solution here

Example 3: Substitution $x = a\sec\theta$ with a definite integral

Find $\displaystyle\int_4^8 \frac{\sqrt{x^2-16}}{x^2}dx.$

Solution

Write the solution here

Example 4: Completing the square and trig sub

Find $\int \frac{x}{\sqrt{x^2-6x+5}}dx$.

Solution

Write the solution here

Example 5: Application - Formula for area of an ellipse

Find the area enclosed by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a, b are positive constants.

Solution

Write the solution here