Review of basic integration rules

Basic antiderivative formulas

$\int k\,dx = kx + C$ where k is a constant

$\int dx = x + C$

$\int kf(x)dx = k\int f(x)dx$

$\int [f(x) \pm g(x)]\,dx = \int f(x)dx \pm \int g(x)dx$

$\int x^n dx = \frac{x^{n+1}}{n+1} + C$, provided $n \neq -1$

$\int \frac{1}{x}dx = \ln|x|$

$\int \cos(x)dx = \sin(x) + C$

$\int \sin(x)dx = -\cos(x) + C$

$\int \sec^2(x)dx = \tan(x) + C$

$\int \sec(x)\tan(x)dx = \sec(x) + C$

$\int \csc^2(x)dx = -\cot(x) + C$

$\int \csc(x)\cot(x)dx = -\csc(x) + C$

$\int \tan(x)dx = -\ln|\cos(x)| + C$

$\int \cot(x)dx = \ln|\sin(x)| + C$

$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$

$\int \csc(x)dx = -\ln|\csc(x) + \cot(x)| + C$

$\int e^x dx = e^x + C$

$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin(\frac{x}{a}) + C$

$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\arctan(\frac{x}{a}) + C$

$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\text{arcsec}(\frac{|x|}{a}) + C$

Formula for finding definite integrals

To find the definite integral of a continuous function f on $[a, b]$, we use the fundamental theorem of calculus

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a).$$

where F is an antiderivative of f. Basically, this says that to find a definite integral of a function, we first find an antiderivative of the function, then evaluate the antiderivative at the upper and lower bounds of integration and find the difference.

Example 1: Find an antiderivative/indefinite integral

Find the antiderivative/indefinite integrals:

1. $\int (x^{3/2} + 2x + 1)dx$
2. $\int (2 - \frac{3}{x^{10}})dx$
3. $\int (\sin(x) - 6\cos(x))dx$
4. $\int \frac{5}{x}dx$
5. $\int \frac{dx}{\sqrt{9 - x^2}}$
6. $\int \frac{dx}{x^2 + 25}$

Solution

Write the solution here

Example 2: Rewrite before integrating

Find the integrals:

1. $\int (x+1)(3x-2)dx$
2. $\int \dfrac{x^4-3x^2+5}{x^4}dx$
3. $\int_1^4 \dfrac{x-2}{\sqrt{x}}dx$
4. $\int_0^{\pi/4} \dfrac{1-\sin^2(x)}{\cos^2(x)}dx$

Solution

Write the solution here

Example 3: Integration by Substitution (u-sub)

1. $\int x\sqrt{x^2+2}dx$
2. $\int_0^4 \frac{1}{\sqrt{2x+1}}dx$
3. $\int \frac{\sin(x)}{\cos^3(x)}dx$
4. $\int \frac{(\ln(x))^2}{x}dx$
5. $\int_0^{\pi/2} e^{\sin(x)}\cos(x)dx$
6. $\int_0^{\ln(5)} \frac{e^x}{1+e^{2x}}dx$

Solution

Write the solution here