Example 1: Find an antiderivative/indefinite integral

Find the antiderivative/indefinite integrals:

1. $\int (x^{3/2} + 2x + 1)dx$
2. $\int (2 - \frac{3}{x^{10}})dx$
3. $\int (\sin(x) - 6\cos(x))dx$
4. $\int \frac{5}{x}dx$
5. $\int \frac{dx}{\sqrt{9-x^2}}$
6. $\int \frac{dx}{x^2+25}$

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1) $$\int \left(x^{3/2} + 2x + 1\right) dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 2\cdot\frac{x^2}{2} + x + C$$

$$= \boxed{\frac{2}{5}x^{\frac{5}{2}} + x^2 + x + C}$$

2) $$\int \left(2 - \frac{3}{x^{10}}\right) dx = \int \left(2 - 3x^{-10}\right) dx = 2x - 3\cdot\frac{x^{-9}}{-9} + C = 2x + \frac{1}{3}x^{-9} + C$$

$$= \boxed{2x + \frac{1}{3x^9} + C}$$

3) $$\int \left[\sin(x) - 6\cos(x)\right] dx = \boxed{-\cos(x) - 6\sin(x) + C}$$

4) $$\int \frac{5}{x} dx = 5\cdot \ln|x| + C$$

5) $$\int \frac{dx}{\sqrt{9-x^2}} = \arcsin\left(\frac{x}{3}\right) + C$$

6) $$\int \frac{dx}{x^2+25} = \frac{1}{5}\arctan\left(\frac{x}{5}\right) + C$$

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Example 2: Rewrite before integrating

Find the integrals:

1. $\int (x+1)(3x-2)dx$
2. $\int \frac{x^4-3x^2+5}{x^4}dx$
3. $\int_1^4 \frac{x-2}{\sqrt{x}}dx$
4. $\int_0^{\pi/4} \frac{1-\sin^2(x)}{\cos^2(x)}dx$

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1. $\int (x+1)(3x-2)\,dx = \int (3x^2+x-2)\,dx = x^3 + \frac{x^2}{2} - 2x + C$

2. $\int (x^4 - 3x^2 + 5)\,x^{-4}\,dx = \int (1 - 3x^{-2} + 5x^{-4})\,dx = x - 3\cdot\frac{x^{-1}}{-1} + 5\cdot\frac{x^{-3}}{-3} + C$

$$= x + 3x^{-1} - \frac{5}{3}x^{-3} + C = \boxed{x + \frac{3}{x} - \frac{5}{3x^3} + C}$$

2nd way: $\int \left(\frac{x^4}{x^4} - \frac{3x^2}{x^4} + \frac{5}{x^4}\right)dx = \int \left(1 - \frac{3}{x^2} + \frac{5}{x^4}\right)dx = \int (1 - 3x^{-2} + 5x^{-4})\,dx$

$$= \boxed{x + \frac{3}{x} - \frac{5}{x^3} + C}$$

3. $\int_1^4 (x-2)\,x^{-\frac{1}{2}}\,dx = \int_1^4 \left(x\cdot x^{-\frac{1}{2}} - 2x^{-\frac{1}{2}}\right)dx = \int_1^4 \left(x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}\right)dx$

$$= \left(\frac{2}{3}x^{\frac{3}{2}} - 2\cdot 2x^{\frac{1}{2}}\right)\Bigg|_1^4 = \left(\frac{2}{3}x^{\frac{3}{2}} - 4x^{\frac{1}{2}}\right)\Bigg|_1^4$$

$$= \left[\frac{2}{3}(4)^{\frac{3}{2}} - 4(4)^{\frac{1}{2}}\right] - \left[\frac{2}{3}(1)^{\frac{3}{2}} - 4(1)^{\frac{1}{2}}\right]$$

$$= \dots$$

$\pi/4$ $\pi/4$ $\pi/4$ $\pi/4$

$= \ldots$

④ $$\int_0^{\pi/4} \frac{1 - \sin^2(x)}{\cos^2(x)}\,dx = \int_0^{\pi/4} \frac{\cos^2(x)}{\cos^2(x)}\,dx = \int_0^{\pi/4} 1\,dx = x\Big|_0^{\pi/4} = \boxed{\frac{\pi}{4}}.$$

Example 3: Integration by Substitution (u-sub)

1. $\int x\sqrt{x^2+2}dx$
2. $\int_0^4 \frac{1}{\sqrt{2x+1}}dx$
3. $\int \frac{\sin(x)}{\cos^3(x)}dx$
4. $\int \frac{(\ln(x))^2}{x}dx$
5. $\int_0^{\pi/2} e^{\sin(x)}\cos(x)dx$
6. $\int_0^{\ln(5)} \frac{e^x}{1+e^{2x}}dx$

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1. $\int \boxed{x}\sqrt{\underbrace{\boxed{x^2+2}}_{u}}\,dx$ Let $u = x^2+2$. $du = 2x\,dx \rightarrow \frac{du}{2} = x\,dx$

$\frac{du}{2}$

$$\frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\int u^{\frac{1}{2}}du = \frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{3}u^{\frac{3}{2}} + C = \frac{1}{3}u^{\frac{3}{2}} + C$$

$$= \boxed{\frac{1}{3}\left(x^2+2\right)^{\frac{3}{2}} + C}$$

2. $\int_0^4 \frac{1}{\sqrt{\underbrace{2x+1}_{u}}}dx$ Let $u = 2x+1$. $du = 2dx \rightarrow dx = \frac{du}{2}$

$\frac{du}{2}$

$$\frac{1}{2}\int_1^9 \frac{1}{\sqrt{u}}du = \frac{1}{2}\int_1^9 u^{-\frac{1}{2}}du = \frac{1}{\cancel{2}}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{\cancel{2}}}\Bigg|_1^9 = (9)^{\frac{1}{2}} - (1)^{\frac{1}{2}} = \boxed{2}$$

3. $\int \frac{\sin(x)}{\cos^3(x)}dx$

$$= \int \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos^2(x)}dx$$

$$= \int \overset{u}{\boxed{\tan(x)}}\cdot\overset{du}{\boxed{\sec^2(x)\,dx}}$$ Let $u = \tan(x)$. $du = \sec^2(x)\,dx$.

$$= \int \boxed{\tan(x)} \cdot \boxed{\sec^2(x)\,dx} \qquad \text{Let } u = \tan(x). \quad du = \sec^2(x)\,dx.$$

$$= \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{\tan^2(x)}{2} + C}.$$

2nd way: $\displaystyle\int \frac{\boxed{\sin(x)\,dx}}{\boxed{\cos^3(x)}_{\to u^3}}$. Let $u = \cos(x)$. $du = -\sin(x)\,dx$

$$= -\int \frac{du}{u^3} = -\int u^{-3}\,du = -\frac{u^{-2}}{-2} + C = \frac{1}{2u^2} + C$$

$$= \boxed{\frac{1}{2\cos^2(x)} + C} = \frac{1}{2}\sec^2(x) + C = \frac{1}{2}\sec^2(x) - \frac{1}{2} + C = \frac{1}{2}(\sec^2(x) - 1) + C$$

$$= \frac{1}{2}\tan^2(x) + C$$

4. $\displaystyle\int \frac{(\overset{u}{\boxed{\ln(x)}})^2}{\boxed{x}}\boxed{dx}_{\to du}$ Let $u = \ln(x)$; $du = \frac{1}{x}\,dx$

$$\int u^2\,du = \frac{u^3}{3} + C = \boxed{\frac{(\ln(x))^3}{3} + C}$$

5. $\displaystyle\int_0^{\pi/2} e^{\overset{u}{\boxed{\sin(x)}}} \overset{du}{\boxed{\cos(x)\,dx}}$ Let $u = \sin(x)$; $du = \cos(x)\,dx$

$$\int_0^1 e^u\,du = e^u \Big|_0^1 = \boxed{e - 1}$$

⑥ $\displaystyle\int_0^{\ln(5)} \frac{\overset{du}{\boxed{e^x\,dx}}}{1 + e^{2x}}$, $e^{2x} \to (e^x)^2$ Let $u = e^x$; $du = e^x\,dx$

$$\int_1^5 \frac{du}{1+u^2} = \arctan(u)\Big|_1^5 = \arctan(5) - \arctan(1)$$

$$= \arctan(5) - \frac{\pi}{4}.$$