Area between curves

Key formulas

- Area A of the region bounded by $y = f(x)$ (top curve), $y = g(x)$ (bottom curve), and the vertical lines $x = a$ and $x = b$ is
$$A = \int_a^b [f(x) - g(x)]\, dx.$$
- For some regions, we need to regard x as a function of y. Area of the region bounded by $x = f(y)$ (right curve), $x = g(y)$ (left curve), and the horizontal lines $y = c$ and $y = d$ is
$$A = \int_c^d [f(y) - g(y)]\, dy.$$
- If two curves intersect at *more than two points*, first find all points of intersection by setting $f = g$ and solve for x (or y). Second, check to see which curve is above (or to the right) the other in each interval determined by these points. Then apply the formula.

Example 1: Find the area of a region between two curves

Sketch the region bounded by $y = x^2 - 1$, $y = -x + 2$, $x = 0$, $x = 1$ and find the area of the region.

Solution

Write the solution here

Example 2: Curves that intersect at more than two points

Find the area of the region bounded by the graphs of $f(x) = x^3 - 3x^2 + 3x$ and $g(x) = x^2$.

Solution

Write the solution here

Example 3: Regard x as a function of y is preferred

Find the area of the region bounded by the graphs of $y = x - 1$ and $y^2 = 2x + 6$.

Solution

Write the solution here

Example 4: An application

The birth rate and death rate of a population is modeled by the functions $b(t) = 2000e^{0.02t}$ and $d(t) = 800e^{0.01t}$, respectively. Find the area between the two curves for on the time interval $[0, 10]$ and explain what this area represents.

Solution

Write the solution here