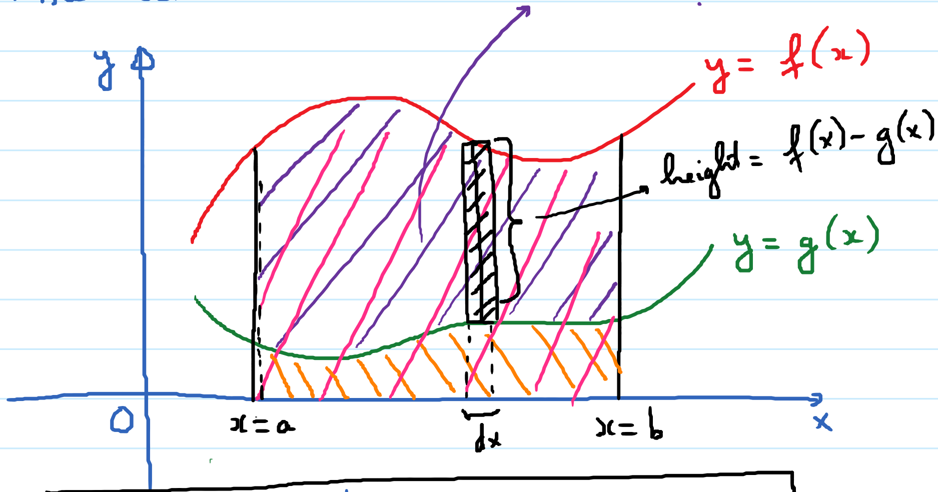
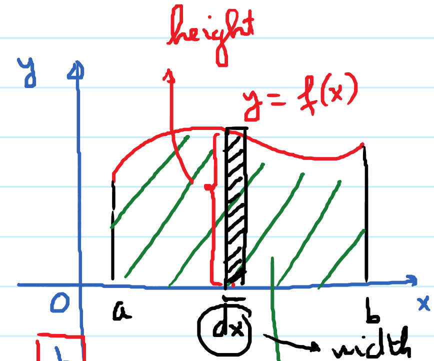
Area between curves

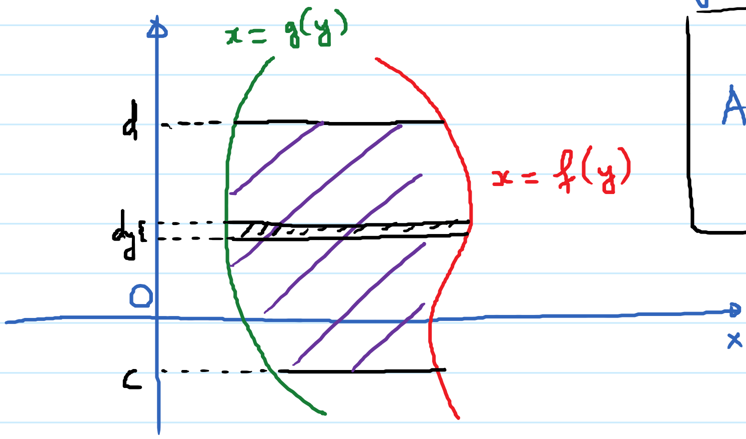




$$\boxed{\int_a^b f(x)\,dx} = \text{Area}$$

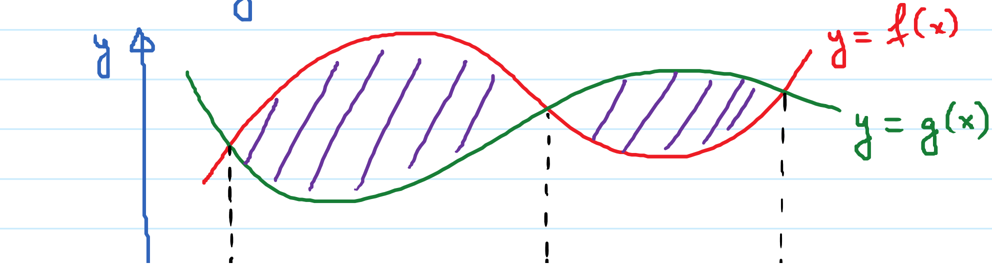
$$\boxed{\text{Area} = A = \int_a^b \Big[\underbrace{f(x)}_{\text{top}} - \underbrace{g(x)}_{\text{bottom}}\Big]\,dx}$$

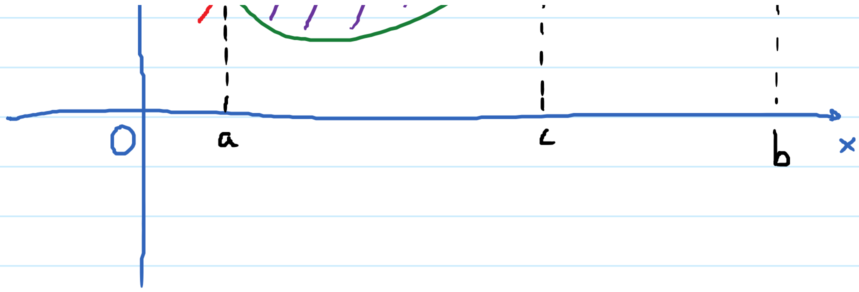
Note: Functions of x in terms of y.



$$\boxed{\text{Area} = A = \int_c^d \Big(\underbrace{f(y)}_{\text{right}} - \underbrace{g(y)}_{\text{left}}\Big)\,dy}$$

* Intersecting curves:





Shaded Area = ?

Strategy: Step 1: Find point(s) of intersection by setting $f(x) = g(x)$ and solve for x.

Step 2: $$\text{Area} = \int_a^c [f(x) - g(x)]\,dx + \int_c^b [g(x) - f(x)]\,dx$$

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Example 1: Find the area of a region between two curves

Sketch the region bounded by $y = x^2 - 1$, $y = -x + 2$, $x = 0$, $x = 1$ and find the area of the region.

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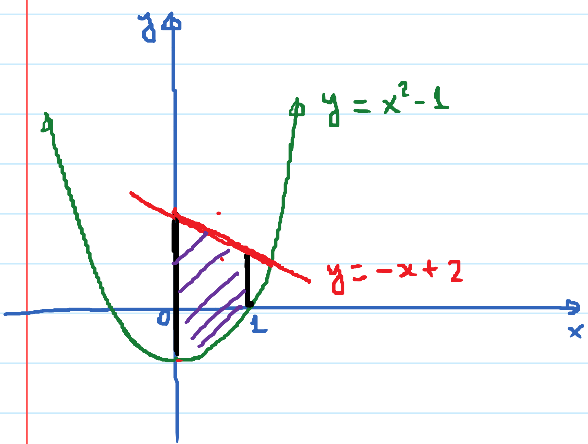

$$\text{Area} = \int_0^1 \left[\underbrace{(-x+2)}_{\text{top}} - \underbrace{(x^2-1)}_{\text{bottom}}\right] dx$$

$$= \int_0^1 \left(-x^2 - x + 3\right) dx = \left(-\frac{x^3}{3} - \frac{x^2}{2} + 3x\right)\Bigg|_0^1$$

$$= \left(-\frac{1}{3} - \frac{1}{2} + 3\right) - 0$$

$$= \boxed{\frac{13}{6}}$$

Example 2: Curves that intersect at more than two points

Find the area of the region bounded by the graphs of $f(x) = x^3 - 3x^2 + 3x$ and $g(x) = x^2$.

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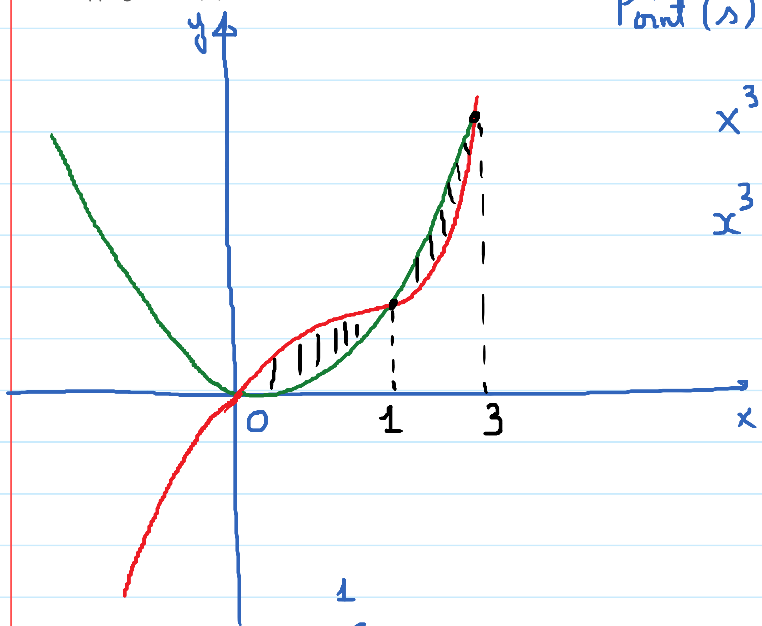

Point(s) of intersection: Set $f(x) = g(x)$

$$x^3 - 3x^2 + 3x = x^2$$

$$x^3 - 4x^2 + 3x = 0$$

$$x(x^2 - 4x + 3) = 0$$

$$x(x-3)(x-1) = 0$$

$$x = 0;\ x = 1;\ x = 3$$

$$\text{Area} = \int_0^1 \left[(x^3 - 3x^2 + 3x) - x^2\right] dx + \int_1^3 \left[x^2 - (x^3 - 3x^2 + 3x)\right] dx$$

$$= \int_0^1 (x^3 - 4x^2 + 3x)\, dx + \int_1^3 (-x^3 + 4x^2 - 3x)\, dx$$

$$= \left(\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right)\Bigg|_0^1 + \left(-\frac{x^4}{4} + \frac{4x^3}{3} - \frac{3x^2}{2}\right)\Bigg|_1^3$$

$$= \frac{5}{12} + \frac{8}{3} = \boxed{\frac{37}{12}}$$

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Example 3: Regard x as a function of y is preferred

Find the area of the region bounded by the graphs of $y = x - 1$ and $y^2 = 2x + 6$.

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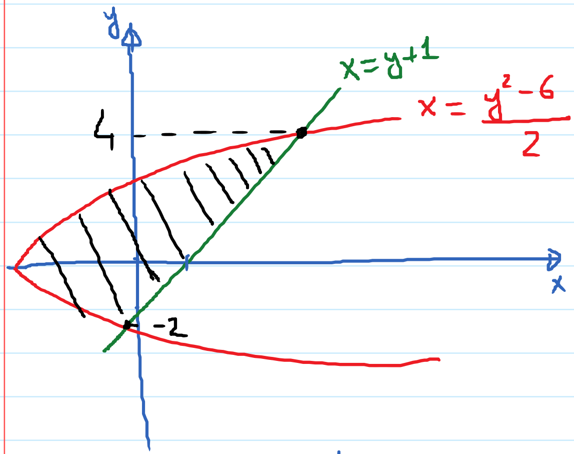
$y = x - 1 \longrightarrow x = y + 1$; $y^2 = 2x + 6 \longrightarrow y^2 - 6 = 2x$

$\longrightarrow x = \dfrac{y^2 - 6}{2}$



Points of intersection:

$$y + 1 = \frac{y^2 - 6}{2}$$

$$2y + 2 = y^2 - 6$$

$$y^2 - 2y - 8 = 0 \longrightarrow (y-4)(y+2) = 0$$

$$\text{Area} = \int_{-2}^{4} \left[(y+1) - \frac{y^2 - 6}{2} \right] dy$$

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Example 4: An application

The birth rate and death rate of a population is modeled by the functions $b(t) = 2000e^{0.02t}$ and $d(t) = 800e^{0.01t}$, respectively. Find the area between the two curves for on the time interval $[0, 10]$ and explain what this area represents.

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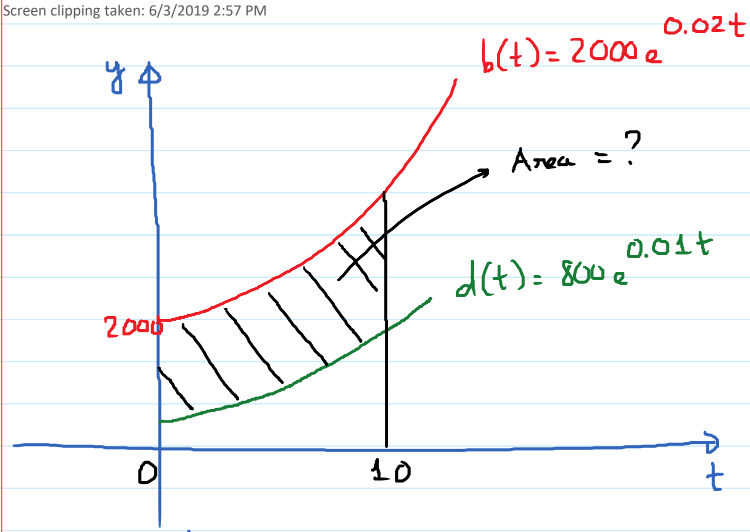

$$\boxed{\int e^{Kt}\,dt = \frac{e^{Kt}}{K} + C}$$

$$\int_0^{10} \left(2000e^{0.02t} - 800e^{0.01t}\right) dt = 2000 \cdot \int_0^{10} e^{0.02t}\,dt - 800 \int_0^{10} e^{0.01t}\,dt$$

$$= 2000 \cdot \left(\frac{e^{0.02t}}{0.02}\right)\Bigg|_0^{10} - 800 \cdot \left(\frac{e^{0.01t}}{0.01}\right)\Bigg|_0^{10}$$

$$= 100000\left(e^{0.2} - 1\right) - 80000\left(e^{0.1} - 1\right)$$