Sequences

Key formulas

A sequence is a list of numbers

$$a_1, a_2, a_3, \dots, a_n, \dots$$

a_1 is called the first term of the sequence, a_2 is the second term, a_n is the **nth term** or the **general term** of the sequence. The entire sequence is denoted by $\{a_n\}$. Quite often, a_n is given by a function $a_n = f(n)$.
If $\lim\limits_{n\to\infty} f(n) = L$ where L is a real number, then we write $\lim\limits_{n\to\infty} a_n = L$ and we say that the sequence **converges** to L. If the limit does not exist, then we say that the sequence **diverges**. If a_n becomes large as n becomes large, then we write $\lim\limits_{n\to\infty} a_n = \infty$ and we say that the sequence **diverges** to infinity.
Squeeze Theorem: If $a_n \le b_n \le c_n$ for all $n > N$ and $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} c_n = L$, then $\lim\limits_{n\to\infty} b_n = L$.
Absolute value theorem: If $\lim\limits_{n\to\infty} |a_n| = 0$, then $\lim\limits_{n\to\infty} a_n = 0$.
A special sequence: If $a_n = r^n$ where r is a constant, then the sequence $\{a_n\}$ converges when $-1 < r \le 1$ and it diverges for all other values of r. Moreover,

$$\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{for } -1 < r < 1 \\ 1 & \text{for } r = 1 \end{cases}$$

A sequence $\{a_n\}$ is **nondecreasing** is $a_1 \le a_2 \le \dots \le a_n \le \dots$; in short, $a_n \le a_{n+1}$ for all $n \ge 1$.
A sequence $\{a_n\}$ is **nonincreasing** if $a_1 \ge a_2 \ge \dots \ge a_n \ge \dots$; in short, $a_n \ge a_{n+1}$ for all $n \ge 1$.
A nondecreasing or nonincreasing sequence is called a **monotonic sequence**.

Example 1: List the terms of a sequence

List the first five terms of the given sequence

1. $a_n = \left(-\dfrac{2}{5}\right)^n$
2. $a_n = \cos\left(\dfrac{n\pi}{2}\right)$
3. $a_n = \dfrac{1}{(n+1)!}$
4. $a_1 = 1, a_2 = 1, a_n = a_{n-1} + a_{n-2}$ for $n \ge 3$.

Solution

Write the solution here

Example 2: Find the nth term of a sequence

Find the formula for the **nth** term of the sequence

1. $1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \ldots$
2. $5, 8, 11, 14, 17, \ldots$

Solution

Write the solution here

Example 3: Find the limit of a sequence

Find the limit of the sequence

1. $a_n = \dfrac{3 + 5n^2}{n + n^2}$
2. $a_n = e^{1/n}$
3. $a_n = n^2 e^{-n}$
4. $a_n = \left(1 + \dfrac{1}{n}\right)^n$

Solution

Write the solution here

Example 4: Squeeze theorem

Find the limit of the sequence $a_n = \dfrac{\sin(2n)}{1+\sqrt{n}}$.

Solution

Write the solution here

Example 5: Absolute value theorem

Find the limit of the sequence

1. $a_n = \dfrac{(-1)^n}{n}$
2. $a_n = \dfrac{\cos(n\pi)}{n^2}$

Solution

Write the solution here

Example 6: Sequence $a_n = r^n$

Determine whether the sequence is convergent of divergent. If it converges, find the limit.

1. $a_n = \dfrac{5^n}{3^n}$
2. $a_n = -3^{-n}$

Solution

Write the solution here

Example 7: Divergent sequences

Explain why the sequence diverges

1. $a_n = (-1)^n$
2. $a_n = \dfrac{n^3}{n+1}$

Solution

Write the solution here

Example 8: Monotonic sequence

Explain why the sequence is monotonic

1. $a_n = \dfrac{3}{n+5}$
2. $a_n = \dfrac{n}{1+n^2}$

Solution

Write the solution here