E.g.2 $a_n = ?$

$$\boxed{a_n = (-1)^{n+1} \cdot \frac{1}{n^2}}$$

$$a_1 = 1 \; ; \; a_2 = -\frac{1}{4} \; , \; a_3 = \frac{1}{9}$$

$$\boxed{a_n = 3n+2}$$

$$a_1 = 5 \; ; \; a_2 = 8 \; , \; a_3 = 11 \; , \; a_4 = 14$$

$$a_n = f(n)$$

$\lim_{n \to \infty} f(n) = L$ and L is a real #

We say that the sequence $\{a_n\}$ converges to L.

If $\lim_{n \to \infty} f(n)$ DNE, then the sequence diverges

When $\lim_{n \to \infty} f(n) = \infty$, the sequence diverges to ∞.

Limit of sequence

E.g. 3

1 $a_n = \dfrac{3 + \boxed{5n^2}}{n + \boxed{n^2}}$ $\rightarrow \dfrac{5\cancel{n^2}}{\cancel{n^2}}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \dfrac{3+5n^2}{n+n^2}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

$= \lim\limits_{n\to\infty} \dfrac{10n}{1+2n}$ $\left(\frac{\infty}{\infty} \text{ form}\right)$

L'Hopital

$= \lim\limits_{n\to\infty} \dfrac{10}{2} = \boxed{5}$

L'Hopital

2 $a_n = e^{1/n}$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} e^{\boxed{1/n}} = e^0 = 1$ $\quad \boxed{1/n} \rightarrow 0$

(3) $a_n = n^2 e^{-n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n^2 e^{-n} = \lim_{n\to\infty} \frac{n^2}{e^n} \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

$$\underset{\text{L'Hopital}}{=} \lim_{n\to\infty} \frac{2n}{e^n} \underset{\text{L'Hopital}}{=} \lim_{n\to\infty} \frac{2}{\boxed{e^n}} = 0.$$

$e^n \to \infty$

(4) $a_n = \left(1 + \frac{1}{n}\right)^n$

$$\boxed{\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e}$$

Squeeze Theorem:

$a_n \le b_n \le c_n$ for all n

and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$

Then $\lim_{n\to\infty} b_n = L$

E.g $a_n = \dfrac{\sin(2n)}{1+\sqrt{n}}$.

$-1 \le \sin(2n) \le 1$ for all n

$$\frac{-1}{1+\sqrt{n}} \le \underbrace{\boxed{\frac{\sin(2n)}{1+\sqrt{n}}}}_{a_n} \le \frac{1}{1+\sqrt{n}}$$

$$\frac{-1}{1+\sqrt{n}} \le a_n \le \frac{1}{1+\sqrt{n}}$$

$$\lim_{n\to\infty}\left(\frac{-1}{1+\sqrt{n}}\right) = \lim_{n\to\infty}\left(\frac{1}{1+\sqrt{n}}\right) = 0$$

So, by Squeeze Theorem, $\boxed{\lim_{n\to\infty} a_n = 0}$

Absolute Value Theorem:

If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$

E.g. $a_n = \dfrac{(-1)^{n+1}}{n}$; $|a_n| = \left|\dfrac{(-1)^{n+1}}{n}\right| = \dfrac{1}{n}$.

0

0

$$a_n = \frac{\cos(n\pi)}{n^2}$$

$$a_1 = -1 \,, \quad a_2 = \frac{1}{4} \,; \quad a_3 = -\frac{1}{9}$$

$$|a_n| = \left| \frac{\cos(n\pi)}{n^2} \right| = \frac{|\cos(n\pi)|}{n^2} = \frac{1}{n^2}$$

$$\lim_{n\to\infty} |a_n| = \lim_{n\to\infty} \frac{1}{n^2} = 0$$

By the Abs. Value Theorem, $\lim\limits_{n\to\infty} a_n = 0$

E.g.6 $$a_n = -3^{-n} = -\frac{1}{3^n} = -\left(\boxed{\frac{1}{3}}\right)^n \to 0$$

$$\lim_{n\to\infty} a_n = 0.$$

E.g.7 $$a_n = (-1)^n$$

$$a_1 = -1 \,; \quad a_2 = 1 \,; \quad a_3 = -1 \,; \quad a_4 = 1$$

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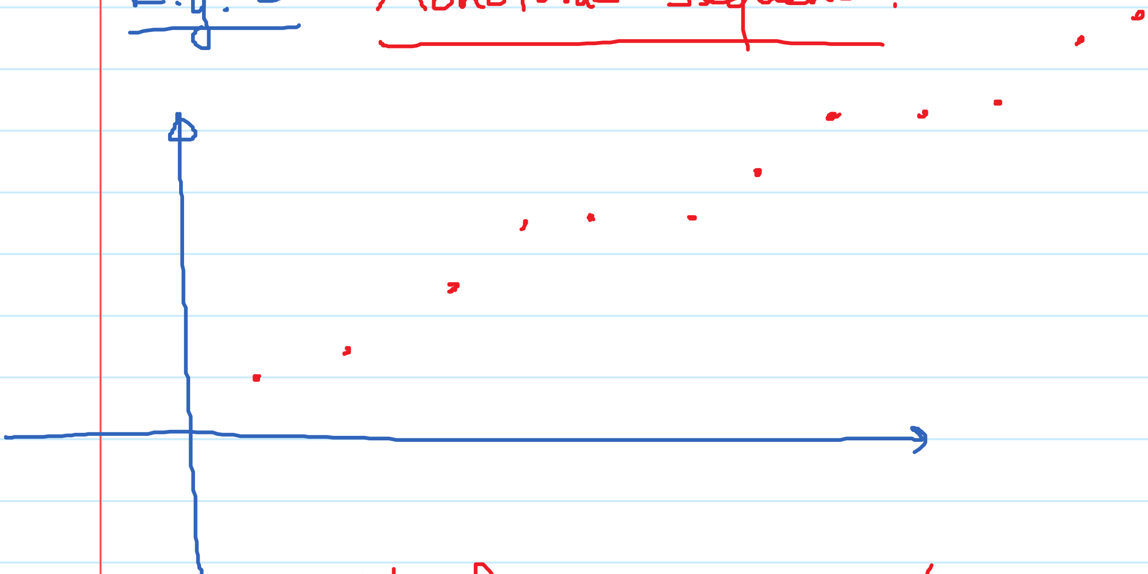
$$a_n = \frac{n^3}{n+1}$$

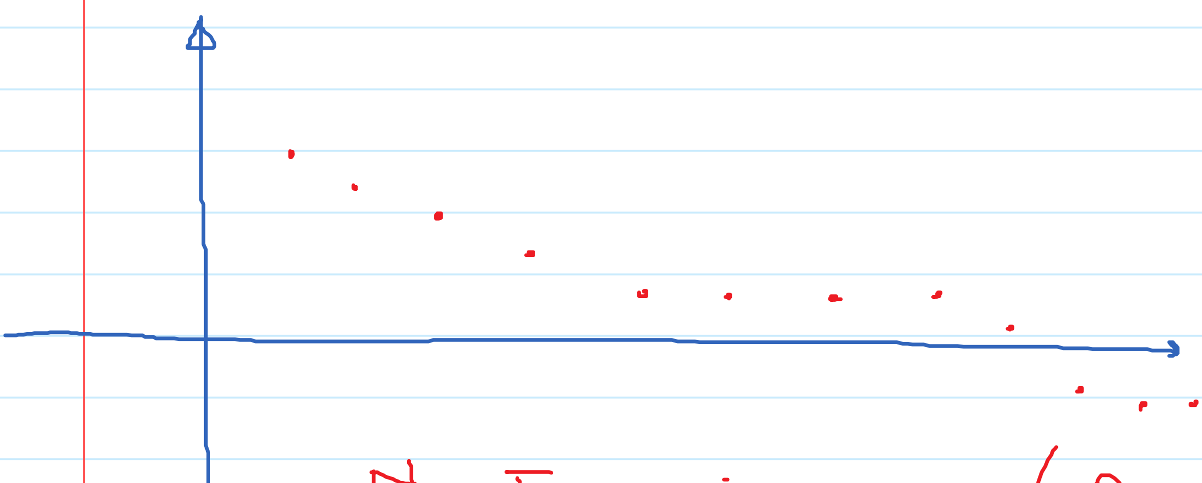
$$\lim_{n\to\infty} \frac{n^3}{n+1} = \infty$$

E.g. 8 Monotonic Sequence :



Non Decreasing sequence (Increasing)

$$a_n \leq a_{n+1}$$



Non Increasing sequence (Decreasing)

$$a_n \geqslant a_{n+1}.$$

E.g. $a_n = \dfrac{3}{n+5}$

$$a_{n+1} = \frac{3}{n+1+5} = \frac{3}{n+6}.$$

$$\frac{3}{n+5} > \frac{3}{n+6} \quad \text{for all } n.$$

$$a_n > a_{n+1}$$

E.g $a_n = \dfrac{n}{1+n^2}$ $\quad a_{n+1} = \dfrac{n+1}{1+(n+1)^2}$

$$f'(n) = \frac{1\cdot(1+n^2) - 2n\cdot n}{(1+n^2)^2}$$

$$= \frac{1+n^2-2n^2}{(1+n^2)^2} = \frac{1-n^2}{(1+n^2)^2}$$

Since $n \geqslant 1$, $\dfrac{1-n^2}{(1+n^2)^2} \leq 0$

So, $f'(n) \leq 0$. So $f(n)$ is decreasing

So, a_n is decreasing.