The integral test and p-series

Key formulas

If the nth term of a series is given by $a_n = f(n)$ where the function $f(x)$ is **positive**, **continuous**, and **decreasing** on the interval $[1, \infty)$ then

$$\sum_{n=1}^{\infty} a_n \text{ and } \int_1^{\infty} f(x)dx$$

either both converge or both diverge.
Note: The lower bound $n = 1$ in the sum and the integral can be replaced by any positive integer $n = N \geq 1$.
Note: The value that the improper integral converges to, in general, is NOT the sum of the series.
p-series test: The p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \dots$$

converges for $p > 1$ and diverges for $0 < p \leq 1$.
Note: The lower bound $n = 1$ in the sum can be replaced by any positive integer $n = N \geq 1$.

Example 1: Using the integral test

Explain why the integral test can be applied to the series. Then apply the test to determine whether the series converges or diverges. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

Solution

Write the solution here

Example 2: Using the integral test

Explain why the integral test can be applied to the series. Then apply the test to determine whether the series converges or diverges.

$$\frac{\ln 2}{2} + \frac{\ln 3}{3} + \frac{\ln 4}{4} + \frac{\ln 5}{5} + \frac{\ln 6}{6} + \dots$$

Solution

Write the solution here

Example 3: Using the integral test

Explain why the integral test cannot be applied to the series.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{\sqrt{n}}$
2. $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\sin n}{n}\right)^2$

Solution

Write the solution here

Example 4: Using the p-series test

Determine whether the series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\pi}}$

2. $1 + \frac{1}{\sqrt[3]{4}} + \frac{1}{\sqrt[3]{9}} + \frac{1}{\sqrt[3]{16}} + \frac{1}{\sqrt[3]{25}} + \dots$

Solution

Write the solution here