The comparison tests

Key formulas

Direct Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with **positive** terms and $0 < a_n \le b_n$ for all $n \ge 1$ (or $n \ge N$ for some positive integer $N \ge 1$), then

In short, if the "larger" series converges, then the "smaller" series must converge. If the "smaller" series diverges, then the "larger" series must diverge.

Limit Comparison Test: Let $\sum a_n$ and $\sum b_n$ be series with **positive** terms, i.e., $a_n > 0$ and $b_n > 0$ for all $n \ge 1$ (or $n \ge N$ for some positive integer $N \ge 1$). If the limit

$$\lim_{n \to \infty} \frac{a_n}{b_n} = L,$$

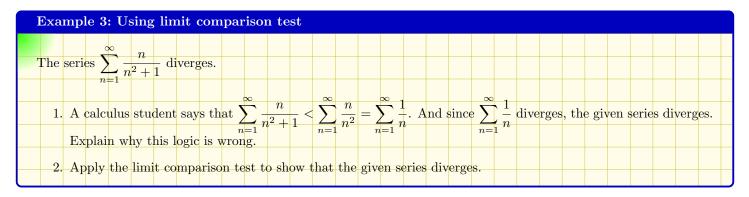
where L is a **finite** and **positive** number, then either both series converge of both series diverge.

Example 1: Using direct comparison test - compare with geometric series Determine whether the series $\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$ converges or diverges.

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Exa	mple 2: Us	ing direct	compariso	n test - c	ompar	re with <i>p</i> -series	
Dete	ermine wheth	er the serie	s converges o	or diverges	5.		
	∞ 1					∞ <u>,</u>	
1.	$\cdot \sum \frac{1}{\sqrt{n}-1}$					2. $\sum \frac{n-1}{n^2/n}$	
	$n=2$ $\sqrt{n-1}$					$n=1$ $n \vee n$	

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Example	e 4: Usin	g lin	nit c	omj	pari	isor	ı te	\mathbf{st}													
										∞		\sqrt{n}	± 2								
Determin	e whether	the	serie	s coi	nverg	ges	or c	liver	ges:		$\frac{1}{2n}$	$\frac{\sqrt{n}}{2}$ +	$\frac{1}{n+2}$	<u>1</u> .							
										n =	2										

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Example 5: Using limit comparison test	
	$\sum_{n=1}^{\infty} n2^n$
Determine whether the series converges or diverges:	$\sum \frac{1}{4n^3+1}$

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