Alternating Series

Key formulas

An **alternating series** is a series of the form

$$\sum_{n=1}^{\infty}(-1)^n a_n \text{ or } \sum_{n=1}^{\infty}(-1)^{n+1} a_n$$

where a_n is a positive number ($a_n > 0$) for each n. The **alternating series test** says that if the following two conditions are met:

1. $\lim_{n\to\infty} a_n = 0$
2. $a_{n+1} \leq a_n$ for all n, i.e., a_n is non-increasing

then the series converges.

Alternating series estimation theorem: Suppose that an alternating series $\sum(-1)^n a_n$ or $\sum(-1)^{n+1} a_n$ converges to a real number s. The **Nth remainder** is $R_N = s - S_N$ where S_N is the sum of the first N terms of the series. The bound for the remainder R_N is given by

$$|R_N| \leq a_{N+1},$$

that is the absolute value of the **Nth remainder** is no more than the $(N+1)$-term, i.e., the "first neglected" term.
We say that a series $\sum a_n$ converges **absolutely** if the "absolute value" series $\sum |a_n|$ converges.
We say that a series $\sum a_n$ converges **conditionally** if $\sum a_n$ converges but the "absolute value" series $\sum |a_n|$ diverges.
Note that the **absolute convergence theorem says that** if the "absolute value" series $\sum |a_n|$ converges, then the orginal series $\sum a_n$ must converge.

Example 1: Using the alternating series test

Explain why the conditions of the alternating series test are met and apply the test to conclude that the series converges

1. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$
2. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{e^n}$

Solution

Write the solution here

Example 2: The alternating series test does not apply

Explain why the alternating series test cannot be applied to the series

1. $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{5n-1}{4n+1}$
2. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\sqrt{n}$

Solution

Write the solution here

Example 3: Using the alternating series estimation theorem

Explain why the conditions of the alternating series test are met and apply the alternating series estimation theorem to determine the number of terms required to approximate the sum of the series with an error of less than 0.001. The series is $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$.

Solution

Write the solution here

Example 4: Absolute convergent series

Explain why the series converges absolutely.

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n\sqrt{n}}$
2. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n^2+1}}$

Solution

Write the solution here

Example 5: Conditionally convergent series

Explain why the series converges conditionally.

1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$
2. $\sum_{n=2}^{\infty}(-1)^{n}\frac{1}{n\ln(n)}$

Solution

Write the solution here