The Ratio and Root Tests

Key formulas

The ratio test: Let $\sum a_n$ be a series. We compute

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|.$$

1. If $L < 1$, then the series $\sum a_n$ **converges absolutely**.
2. If $L > 1$ or $L = \infty$, then the series $\sum a_n$ **diverges**.
3. If $L = 1$, then the test is inconclusive. In this case, the test fails and we cannot draw any conclusion about the convergence or divergence of the series

The root test: Let $\sum a_n$ be a series. We compute

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} |a_n|^{\frac{1}{n}}.$$

1. If $L < 1$, then the series $\sum a_n$ **converges absolutely**.
2. If $L > 1$ or $L = \infty$, then the series $\sum a_n$ **diverges**.
3. If $L = 1$, then the test is inconclusive. In this case, the test fails and we cannot draw any conclusion about the convergence or divergence of the series

Example 1: Using the ratio test

Determine whether the series converges absolutely or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{3^n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{n!}$

Solution

Write the solution here

Example 2: The ratio test fails

Explain why the ratio test fails for the series $\sum_{n=1}^{\infty}(-1)^n\frac{\sqrt{n}}{n+1}$. Determine whether the series converges conditionally, converges absolutely or diverges.

Solution

Write the solution here

Example 3: Using the root test

Determine whether the series converges absolutely or diverges.

1. $\sum_{n=1}^{\infty}\left(\frac{2n}{n+1}\right)^n$
2. $\sum_{n=1}^{\infty}\frac{(-2)^n}{n^n}$

Solution

Write the solution here

Example 4: Make a series converge

Find the values of x for which the series converges

1. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}$

Solution

Write the solution here