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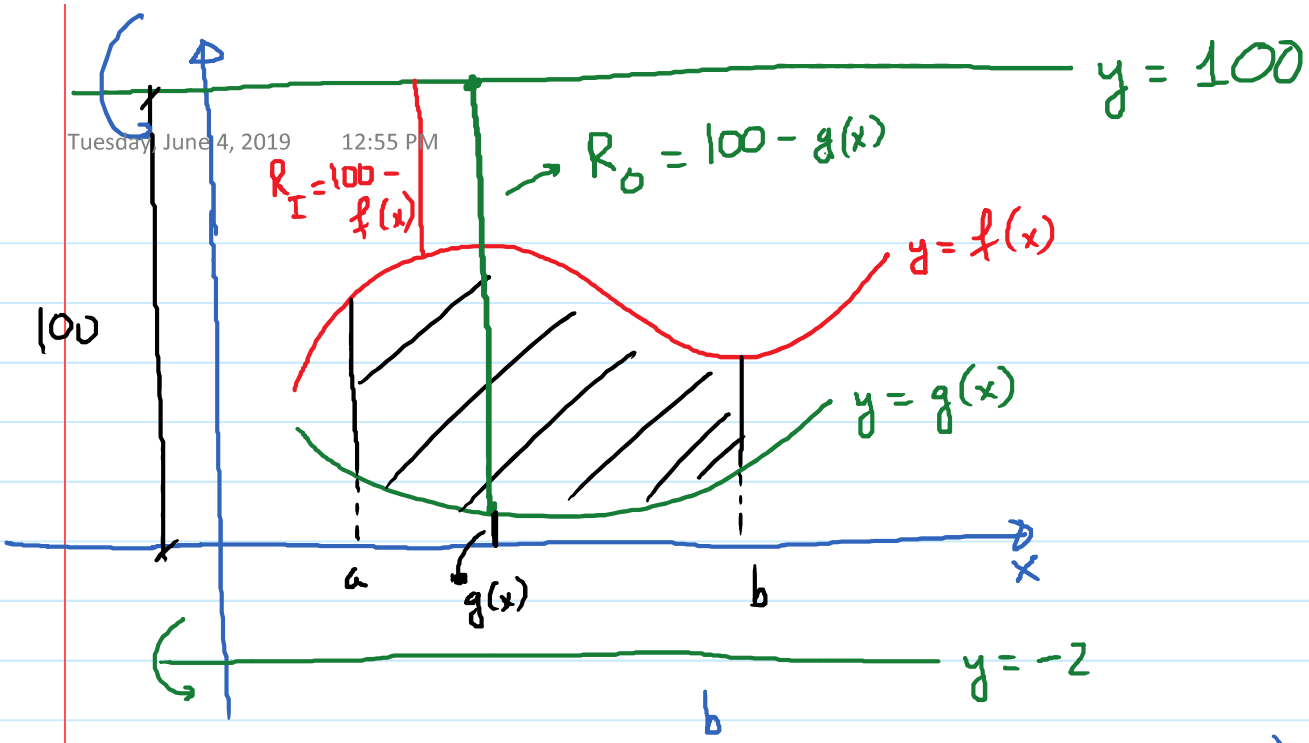
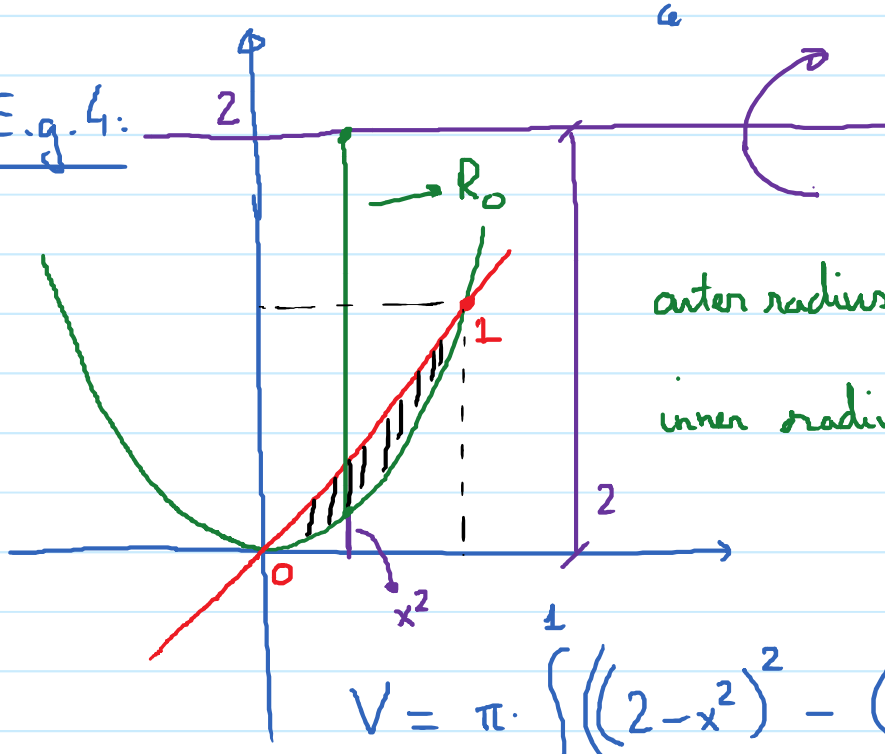


Rotate about $y = -2$: $V = \pi \cdot \int_a^b \left((f(x)+2)^2 - (g(x)+2)^2 \right) dx$

Rotate about $y = 100$: $V = \pi \cdot \int_a^b \left((100-g(x))^2 - (100-f(x))^2 \right) dx$.

E.g. 4:



outer radius $= 2 - x^2$

inner radius $= 2 - x$

$V = \pi \cdot \int_0^1 \left((2-x^2)^2 - (2-x)^2 \right) dx$

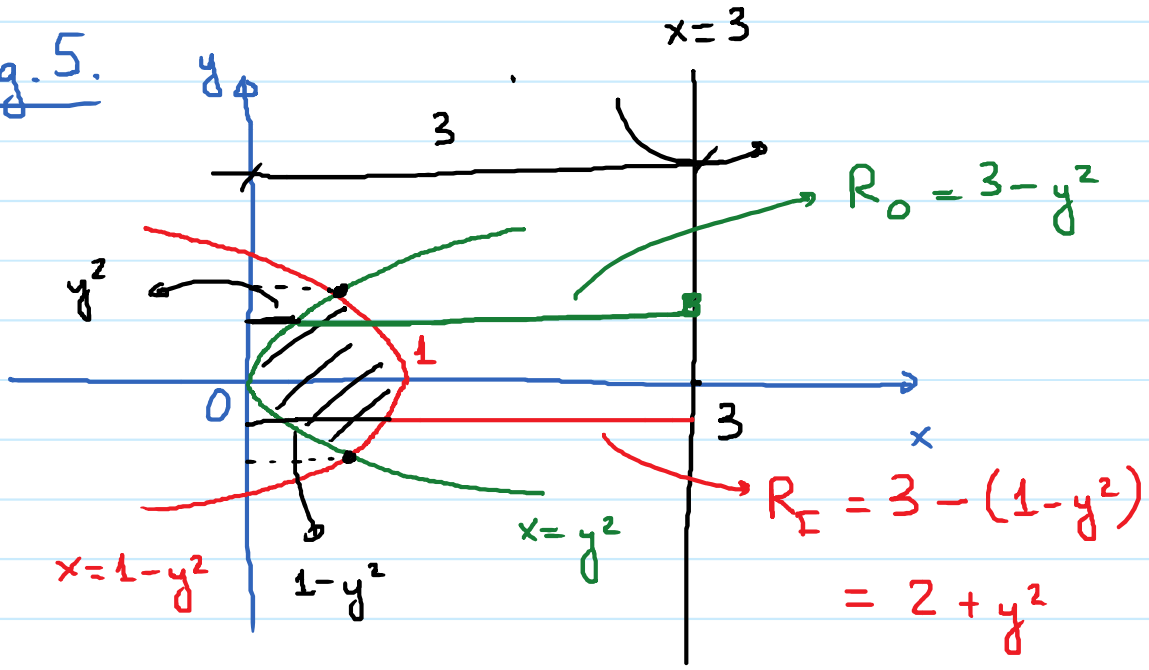
$$V = \pi \cdot \int_0^1 \left(\left(4 - 4x^2 + x^4\right) - \left(4 - 4x + x^2\right) \right) dx$$

$$= \pi \cdot \int_0^1 \left(x^4 - 5x^2 + 4x\right) dx$$

$$= \pi \cdot \left(\frac{x^5}{5} - 5\frac{x^3}{3} + 2x^2 \right) \Big|_0^1$$

$$= \pi \left(\frac{1}{5} - \frac{5}{3} + 2 \right) = \dots$$

E.g. 5.



outer Radius $= 3 - y^2$; inner radius $= 2 + y^2$

Points of intersection: $y^2 = 1 - y^2 \rightarrow 2y^2 = 1$

$$\rightarrow y^2 = \frac{1}{2} \rightarrow y = \pm \frac{1}{\sqrt{2}}$$

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$$V = \pi \int_{-\frac{1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \left[\left(3 - y^2\right)^2 - \left(2 + y^2\right)^2 \right] dy.$$

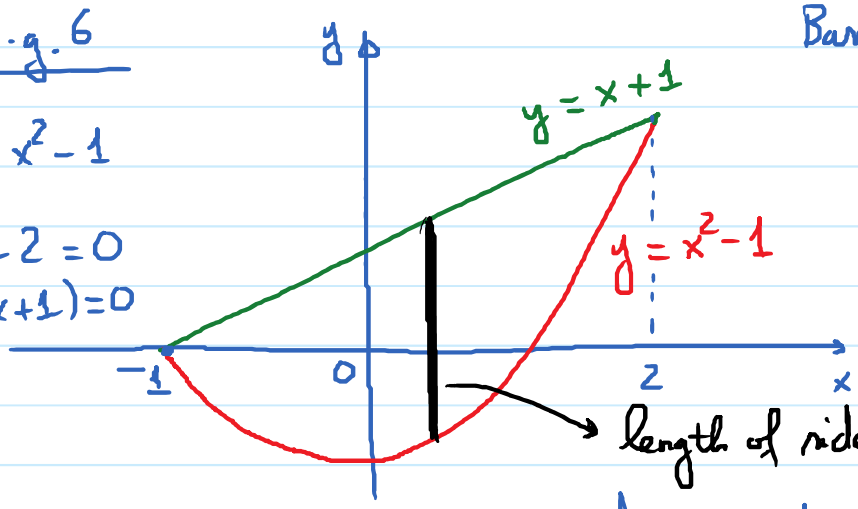
E.g. 6

Base of object on xy-plane

$x+1 = x^2 - 1$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$



length of side of the square

$\text{side} = x+1-(x^2-1)$

$= -x^2+x+2$

Cross-sections are squares

$\rightarrow$ cross-section area $= (\text{side})^2$

$= (-x^2+x+2)^2$

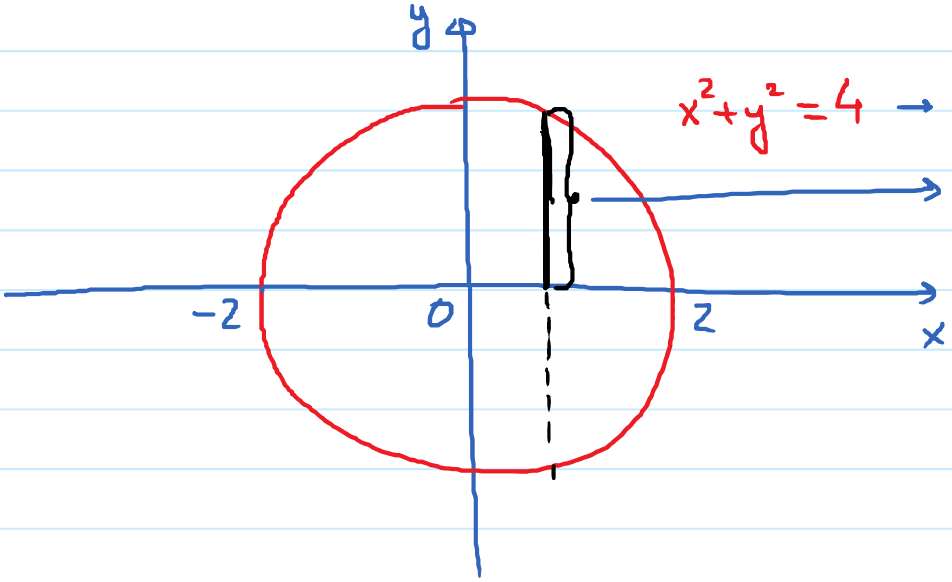
$$V = \int_{-1}^{2} (-x^2+x+2)^2 \, dx$$

E.g. 7.

cross section area = area of a semi-disk

$$= \frac{\pi \cdot (\text{Radius})^2}{2}.$$

$\rightarrow$ Find radius.



$x^2+y^2=4 \rightarrow y^2=4-x^2$

$y=\sqrt{4-x^2}$

Radius $=\sqrt{4-x^2}$.

$$V=\int_{-2}^{2}\frac{\pi\cdot\left(\sqrt{4-x^2}\right)^2}{2}\,dx$$

$$=\frac{\pi}{2}\int_{-2}^{2}\left(4-x^2\right)dx=\frac{\pi}{2}\cdot\left(4x-\frac{x^3}{3}\right)\Bigg|_{-2}^{2}$$

Volume of object whose cross sections are given is:

$$V=\int_a^b A(x)\,dx$$

(if cross section perpendicular to x-axis)

$A(x)$ → formula for cross-section area.

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$$V = \int_c^d A(y)\,dy$$ (cross-sections perpendicular to y-axis)

$A(y)$ → formula for cross-section area.