Volume - The Disk and the Washer Method

Key formulas

- The basic formula (**disk method**) to find the volume of the object obtained by revolving a region bounded by a curve about an axis (horizontal or vertical) is

$$\text{Volume} = \pi \int_{\text{lower bound}}^{\text{upper bound}} (\text{Radius})^2 \cdot \text{thickness}$$

  where the radius is measured from the axis of rotation to the curve and it is a function of x (or y), and the thickness is dx (or dy).

  If we rotate the region bounded by the curve $y = R(x)$, $a \leq x \leq b$ and the x-axis ($y = 0$) about the x-axis, the above formula translates to

$$\text{Volume} = V = \pi \int_a^b (R(x))^2 \, dx.$$

  If we rotate the region bounded by the curve $x = R(y)$, $c \leq y \leq d$ and the y-axis ($x = 0$) about the y-axis, the above formula translates to

$$\text{Volume} = V = \pi \int_c^d (R(y))^2 \, dy.$$

- The basic formula (**washer method**) to find the volume of the object obtained by revolving a region bounded by two curves about an axis is

$$\text{Volume} = \pi \int_{\text{lower bound}}^{\text{upper bound}} \left[(\text{outer radius})^2 - (\text{inner radius})^2\right] \cdot \text{thickness}$$

  where the outer radius is measured from the axis of rotation to the curve that is above (or on the right), the inner radius is measured from the axis of rotation to the curve that is below (or on the left), both radii are functions of x (or y). The thickness is either dx or dy.

- The basic formula to find the volume of an object whose cross section area is known as a function of x or y is

$$\text{Volume} = \int_{\text{lower bound}}^{\text{upper bound}} (\text{cross section area}) \cdot \text{thickness}.$$

  If the cross sections are perpendicular to the x-axis and the cross section area is given by $A(x)$, $a \leq x \leq b$ the above formula is translated to

$$\text{Volume} = \int_a^b A(x) dx$$

  If the cross sections are perpendicular to the x-axis and the cross section area is given by $A(y)$, $c \leq x \leq d$ the above formula is translated to

$$\text{Volume} = \int_c^d A(y) dy$$

  To find the cross section area, it is useful to remember the formula for the area of some basic shapes, for example,

$$A_{\text{square}} = (\text{side})^2, A_{\text{equilateral triangle}} = (\text{side})^2 \cdot \frac{\sqrt{3}}{4}, A_{\text{circle}} = \pi \cdot (\text{Radius})^2.$$

Example 1: Apply the disk method-revolve about x-axis

Find the volume of the solid formed by revolving the region bounded by the curve $y = \sqrt{x}$, $1 \leq x \leq 4$ and the x-axis ($y = 0$) about the x-axis.

Solution

Write the solution here

Example 2: Apply the disk method-revolve about y-axis

Find the volume of the solid formed by revolving the region bounded by the curves $y = \ln(x)$, $y = 1$, $y = 2$, $x = 0$ (y-axis) about the y-axis.

Solution

Write the solution here

Example 3: Apply the washer method

Find the volume of the solid formed by revolving the region in the first quadrant bounded by the curves $y = x^3$ and $y = x$ about the x-axis.

Solution

Write the solution here

Example 4: Revolve about a line that is not a coordinate axis

Find the volume of the solid formed by revolving the region bounded by $y = x$, $y = x^2$ about the line $y = 2$.

Solution

Write the solution here

Example 5: Revolve about a line that is not a coordinate axis

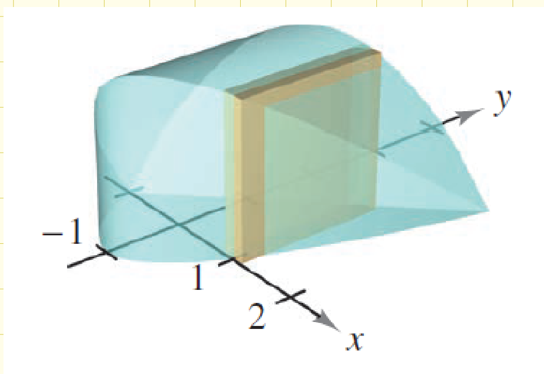
Set up the integral (no need to evaluate) to find the volume of the solid formed by revolving the region bounded by $x = y^2$, $x = 1 - y^2$ about the line $x = 3$.

Solution

Write the solution here

Example 6: Volume of solid with given cross sections

Find the volume of the solid whose base is the region bounded by the curves $y = x + 1$ and $y = x^2 - 1$ and whose cross sections perpendicular to the x-axis are squares.

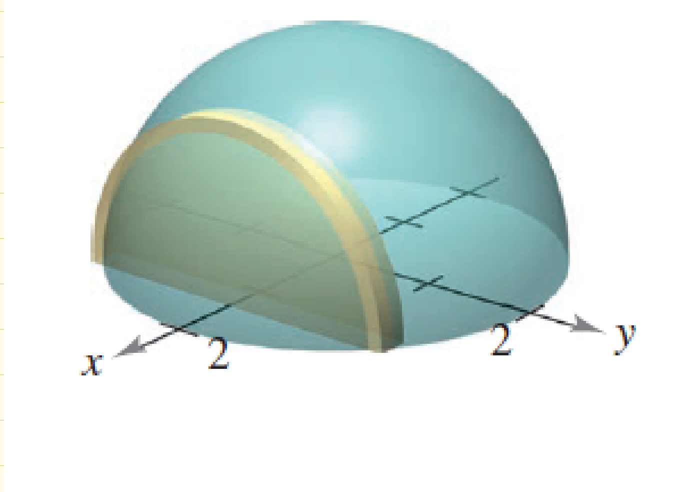


Solution

Write the solution here

Example 7: Volume of solid with given cross sections

Set up the integral (no need to evaluate) to find the volume of the solid whose base is the region bounded by the circle $x^2 + y^2 = 4$ and and whose cross sections perpendicular to the x-axis are semicircles.



Solution

Write the solution here