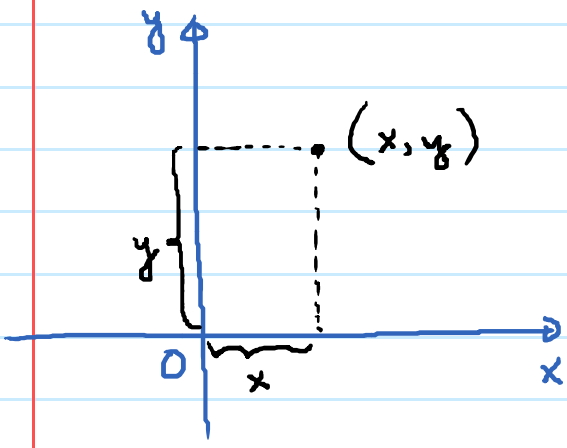
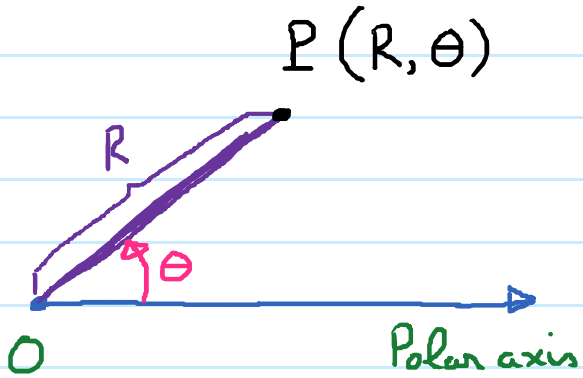
Polar Coordinates

Wednesday, July 3, 2019 12:00 PM



Rectangular Coordinate System

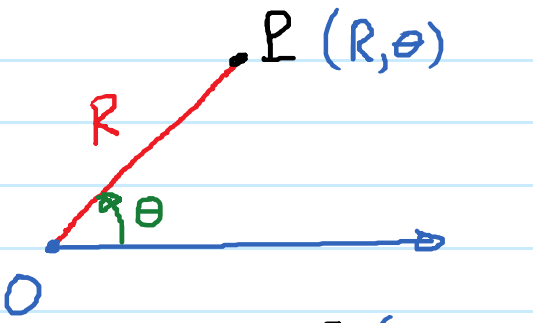


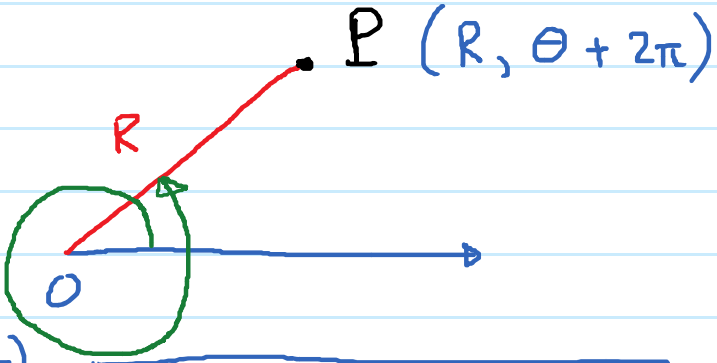
Polar coordinate system

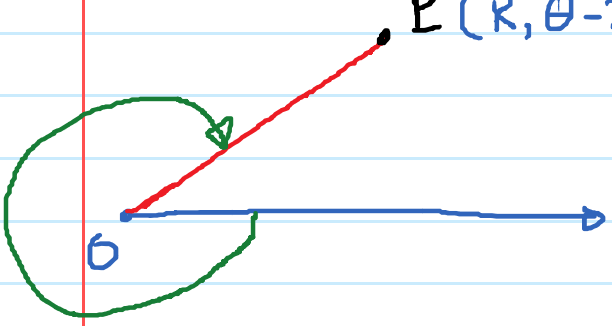
R = distance from O to P

θ = angle measured counter-clockwise from the polar axis to ray OP.

Note: Polar coordinates of a point are not unique.







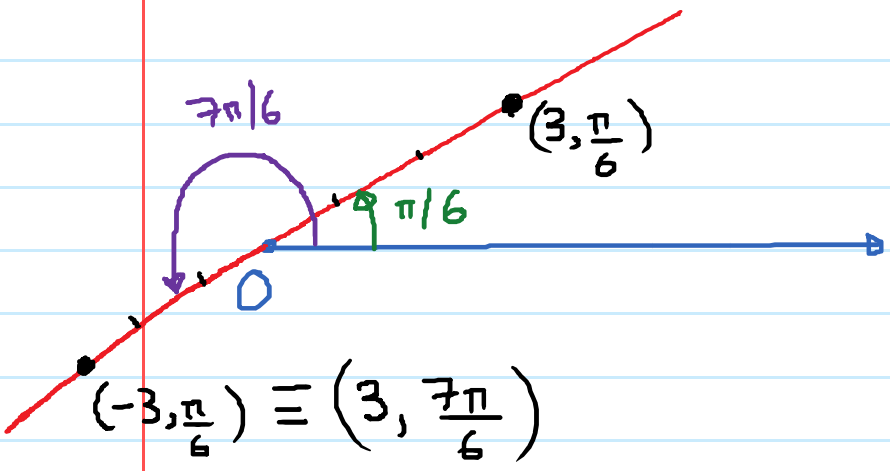
$$(R,\theta) \equiv (R, \theta + n \cdot 2\pi)$$

where n is any integer

E.g. 1:

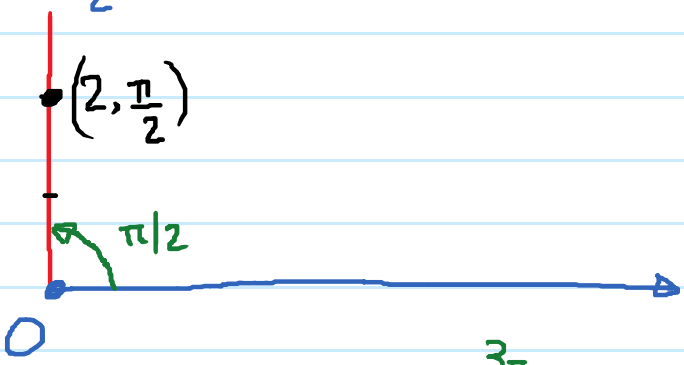
(4) Plot the point with polar coordinates $(-3, \frac{\pi}{6})$.

(Case where $R < 0$)

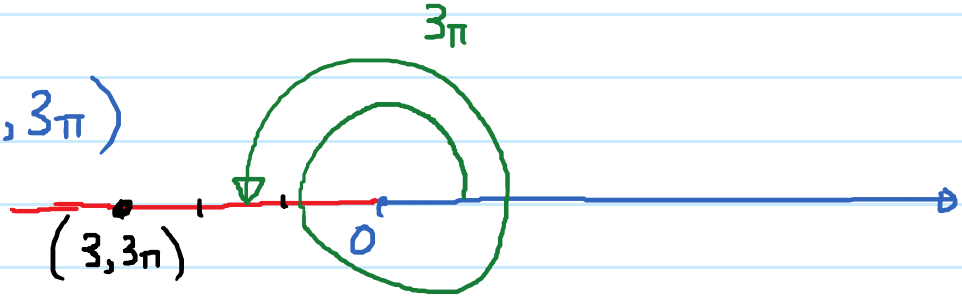


Note: If $R < 0$, then $(R, \theta) \equiv (-R, \theta + \pi)$

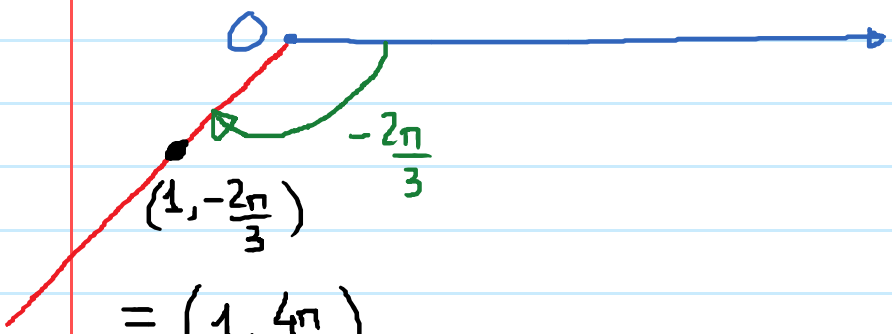
(1) $(2, \frac{\pi}{2})$



(2) $(3, 3\pi)$



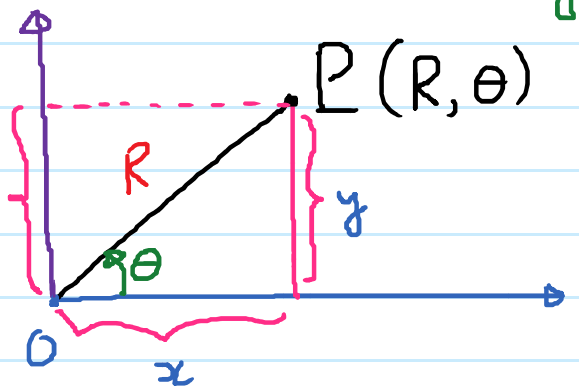
③ $\left(1, -\frac{2\pi}{3}\right)$



$\equiv \left(1, \frac{4\pi}{3}\right)$

Convert between polar and rectangular coordinates.

Polar ⟶ Rectangular.



$$x = R\cos\theta \quad ; \quad y = R\sin\theta$$

E.g. 2 ① $\left(2, \frac{3\pi}{4}\right)$ a point with polar coord.

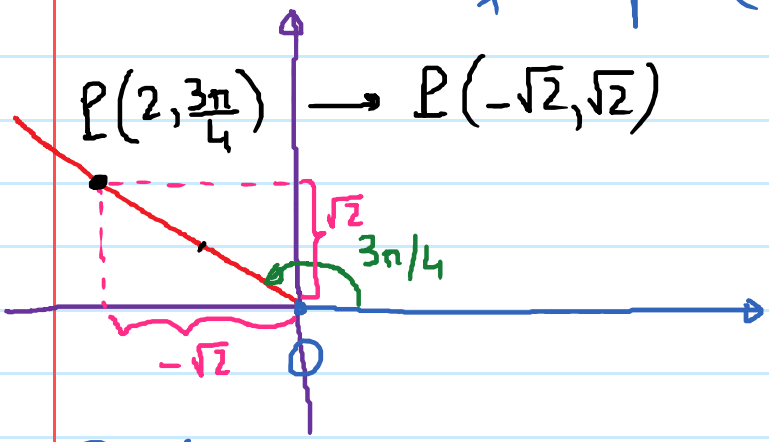
Find rect. coord.

$x = 2\cos\left(\frac{3\pi}{4}\right)$; $y = 2\sin\left(\frac{3\pi}{4}\right)$

$x = 2\left(-\frac{\sqrt{2}}{2}\right)$; $y = 2\left(\frac{\sqrt{2}}{2}\right)$

$x = -\sqrt{2}$ $y = \sqrt{2}$

Rect. coord. of the pt: $(-\sqrt{2}, \sqrt{2})$



② $\left(-2, \frac{5\pi}{3}\right)$ → Rect. coord.

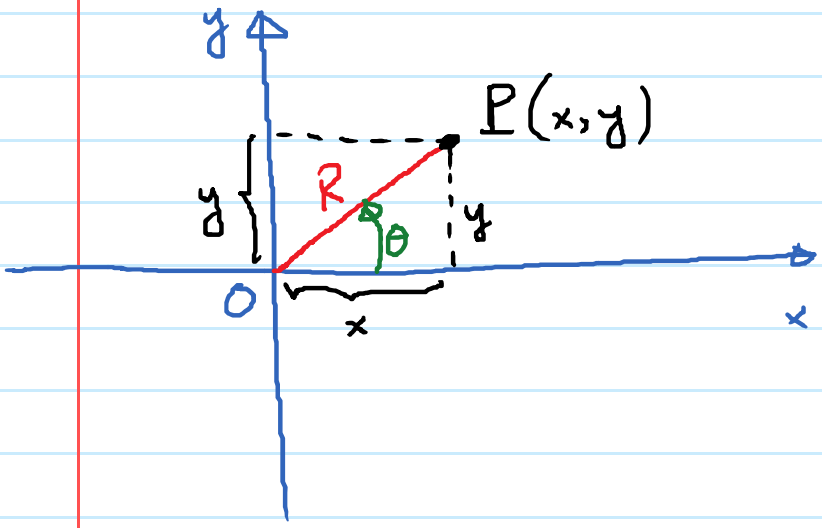
$x = R\cos\theta$; $y = R\sin\theta$.

$x = (-2)\cos\left(\frac{5\pi}{3}\right)$; $y = (-2)\sin\left(\frac{5\pi}{3}\right)$

$x = (-2)\left(\frac{1}{2}\right)$; $y = (-2)\left(-\frac{\sqrt{3}}{2}\right)$

$x = -1$ $y = \sqrt{3}$ → $(-1, \sqrt{3})$

Rectangular → Polar.



$$R = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

$\theta = \arctan\left(\frac{y}{x}\right)$ if point is in QI or QIV

$\theta = \arctan\left(\frac{y}{x}\right) + \pi$ if point is in QII or QIII.

E.g.3: Given Rectangular coordinates of a point, find the polar coordinates:

(1) $(3\sqrt{3}, 3)$ (in QI)

($3\sqrt{3}$ = x, 3 = y)

$$R = \sqrt{(3\sqrt{3})^2 + (3)^2} = \sqrt{36} = 6$$

$$\theta = \arctan\left(\frac{3}{3\sqrt{3}}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

Polar coord. $\left(6, \frac{\pi}{6}\right)$

(2) $(-1, \sqrt{3})$ (QII)

$x \quad y$

$$R = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$$

$$\theta = \arctan\left(\frac{\sqrt{3}}{-1}\right) + \pi$$

$$= \arctan(-\sqrt{3}) + \pi$$

$$\theta = -\frac{\pi}{3} + \pi = \frac{2\pi}{3}$$

$\left(2, \frac{2\pi}{3}\right)$

(3) $(2, -2)$ (QIV)

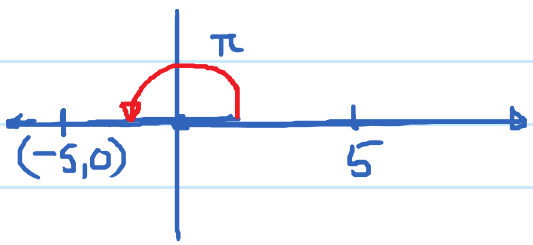
$$R = \sqrt{(2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$\theta = \arctan\left(\frac{y}{x}\right) = \arctan\left(\frac{-2}{2}\right) = \arctan(-1) = -\frac{\pi}{4}.$$

$\longrightarrow$ Polar coord: $\left(2\sqrt{2}, -\frac{\pi}{4}\right)$

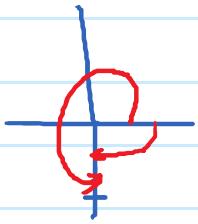
(4) $(-5, 0)$

$$R = \sqrt{(-5)^2 + (0)^2} = 5$$



$\theta = \pi \longrightarrow (5, \pi)$

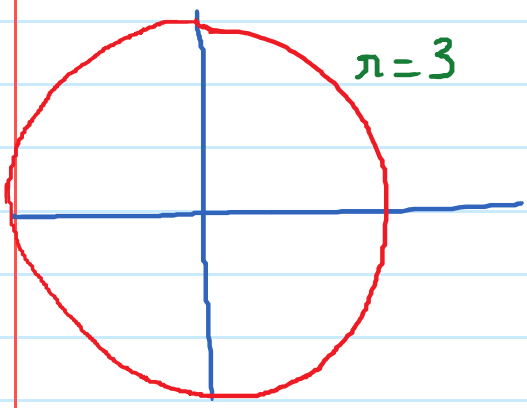
$(0, -5)$



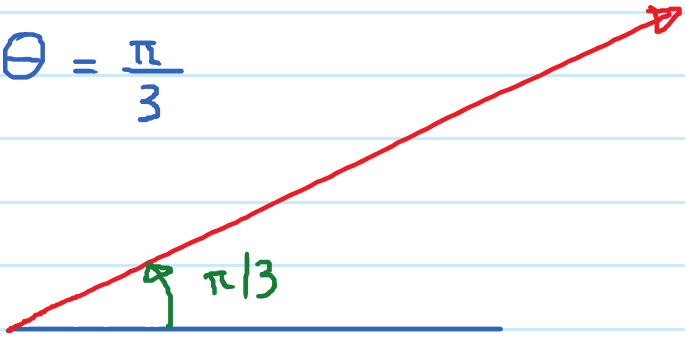
$R = 5; \ \theta = \frac{3\pi}{2}$

E.g. 4 Sketch the graph of the curve with given polar equation.

(1) $r = 3$ ⟶ circle centered at origin and radius = 3



(2) $\theta = \frac{\pi}{3}$



(3) $r = \cos(2\theta)$