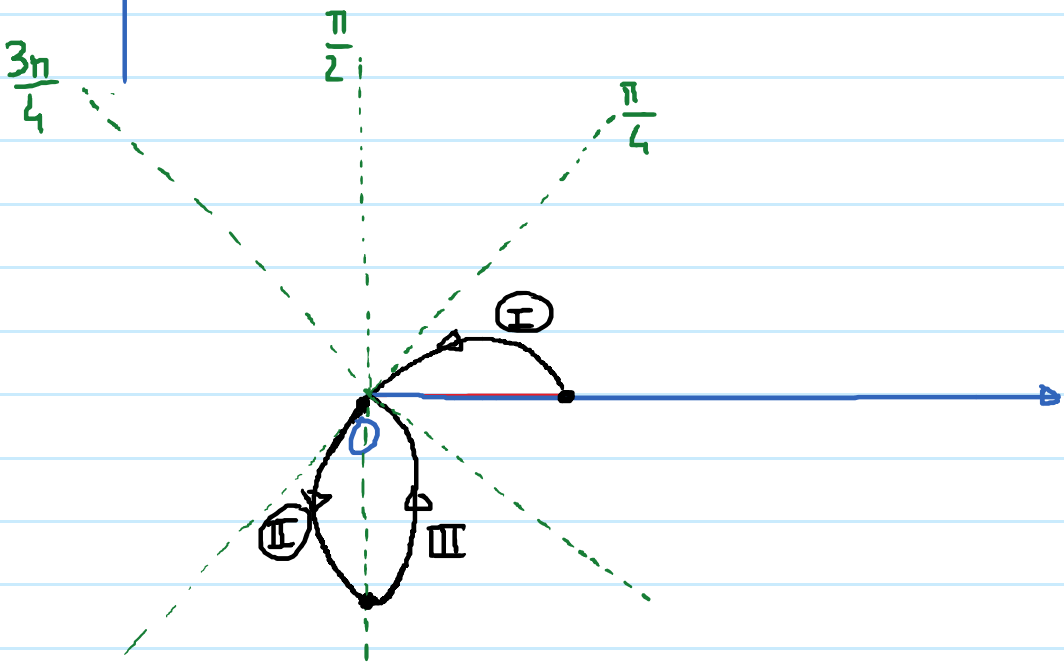
	θ	r
I	$0 \le \theta \le \frac{\pi}{4}$	$r = 1$ decreases 0
II	$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$	$r = 0$ decreases to -1
III	$\frac{\pi}{2} \le \theta \le \frac{3\pi}{4}$	$r = -1$ increases to 0



E.g. 5. Convert between a polar equation and a rectangular equation:

(1) $r = 5\cos\theta$. $\longrightarrow$ rect. equation.

$\longrightarrow r^2 = 5r\cos\theta$

$r^2 = x^2 + y^2$, $r\cos\theta = x$

$\longrightarrow \boxed{x^2 + y^2 = 5x}$

$x = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$

$\tan\theta = \frac{y}{x}$

(2) $r = 2\csc\theta$.

$r = \frac{2}{\sin\theta} \longrightarrow r\sin\theta = 2 \longrightarrow \boxed{y = 2}$

(3) $r = \cot\theta\csc\theta$.

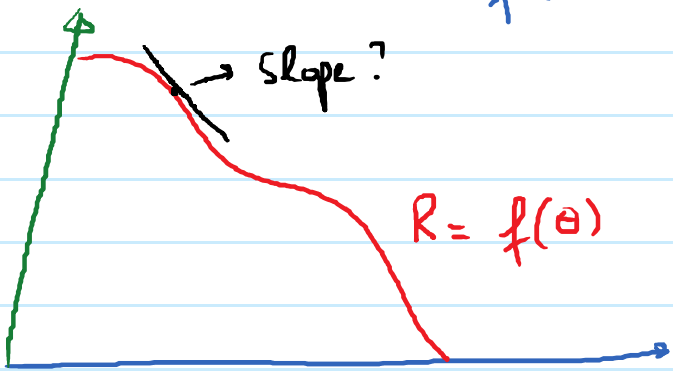
$r = \frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta} \longrightarrow r\sin\theta = \frac{\cos\theta}{\sin\theta}$

$\longrightarrow r\sin\theta = \frac{1}{\tan\theta} \longrightarrow y = \frac{1}{\frac{y}{x}} = \frac{x}{y}$

$\longrightarrow \boxed{y^2 = x}$.

Slopes of tangent lines.

Polar curve : $R = f(\theta)$



$$\text{Slope} = \frac{dy}{dx}$$

* Turn the polar equation $R = f(\theta)$ into a set of 2 parametric equations:

$$x = R\cos\theta = f(\theta)\cos\theta$$

$$y = R\sin\theta = f(\theta)\sin\theta$$

* $$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{d}{d\theta}\left[f(\theta)\sin\theta\right]}{\frac{d}{d\theta}\left[f(\theta)\cos\theta\right]}$$

$$= \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

E.g.6 $r = 2(1 - \sin\theta)$

Find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$x = r\cos\theta = 2(1 - \sin\theta)\cos\theta$$

$$y = r\sin\theta = 2(1 - \sin\theta)\sin\theta$$

$$\frac{dx}{d\theta} = 2 \cdot \left[(1 - \sin\theta)\cdot(-\sin\theta) + (-\cos\theta)\cdot\cos\theta\right]$$

$$= 2\left[-\sin\theta + \sin^2\theta - \cos^2\theta\right]$$

$$\frac{dy}{d\theta} = 2 \cdot \left[(1 - \sin\theta)\cdot\cos\theta + (-\cos\theta)\cdot\sin\theta\right]$$

$$= 2\left[\cos\theta - \sin\theta\cos\theta - \sin\theta\cos\theta\right]$$

$$= 2\left[\cos\theta - 2\sin\theta\cos\theta\right]$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos\theta - 2\sin\theta\cos\theta}{-\sin\theta + \sin^2\theta - \cos^2\theta}$$

Slope of tangent line to the curve at $\theta = 0$: $\frac{1}{-1} = -1$.

(2) $r = 3\cos\theta$; $0 \le \theta \le 2\pi$.

Tangent line is horizontal : Slope $= 0$

$$\text{Slope} = \frac{dy/d\theta}{dx/d\theta}$$

$$\text{Slope} = 0 \rightarrow \frac{dy}{d\theta} = 0$$

$$y = r\sin\theta = 3\cos\theta\sin\theta$$

$$\frac{dy}{d\theta} = 3\cdot\left[\cos\theta\cdot\cos\theta - \sin\theta\cdot\sin\theta\right]$$

$$= 3\left[\cos^2\theta - \sin^2\theta\right]$$

Set $3\left[\cos^2\theta - \sin^2\theta\right] = 0$

$$(\cos\theta - \sin\theta)(\cos\theta + \sin\theta) = 0$$

$\cos\theta - \sin\theta = 0$; $\cos\theta + \sin\theta = 0$

$\tan\theta = 1$; $\tan\theta = -1$

$\theta = \frac{\pi}{4}; \frac{5\pi}{4}$; $\theta = \frac{3\pi}{4}; \frac{7\pi}{4}$.

Tangent line is vertical : Slope = undefined

$$\text{Slope} = \frac{dy/d\theta}{dx/d\theta}$$

For slope to be undefined: $\frac{dx}{d\theta} = 0$

$$x = r\cos\theta = 3\cos\theta\cos\theta = 3\cos^2\theta$$

$$\frac{dx}{d\theta} = 6\cos\theta\cdot(-\sin\theta) = 0$$

$$\cos\theta = 0 \quad ; \quad \sin\theta = 0$$

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2} \quad ; \quad \theta = 0, \pi.$$