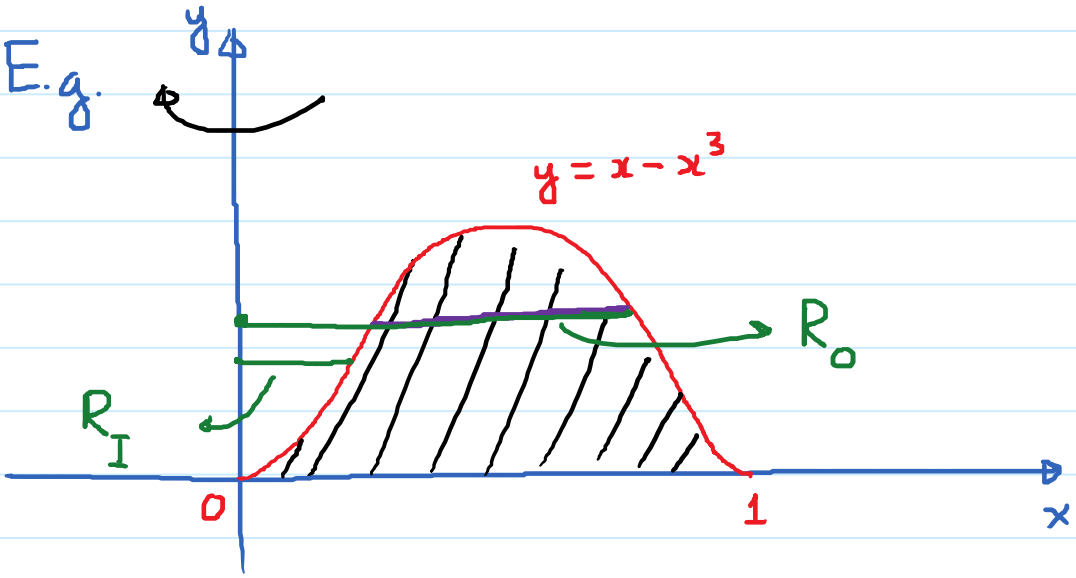
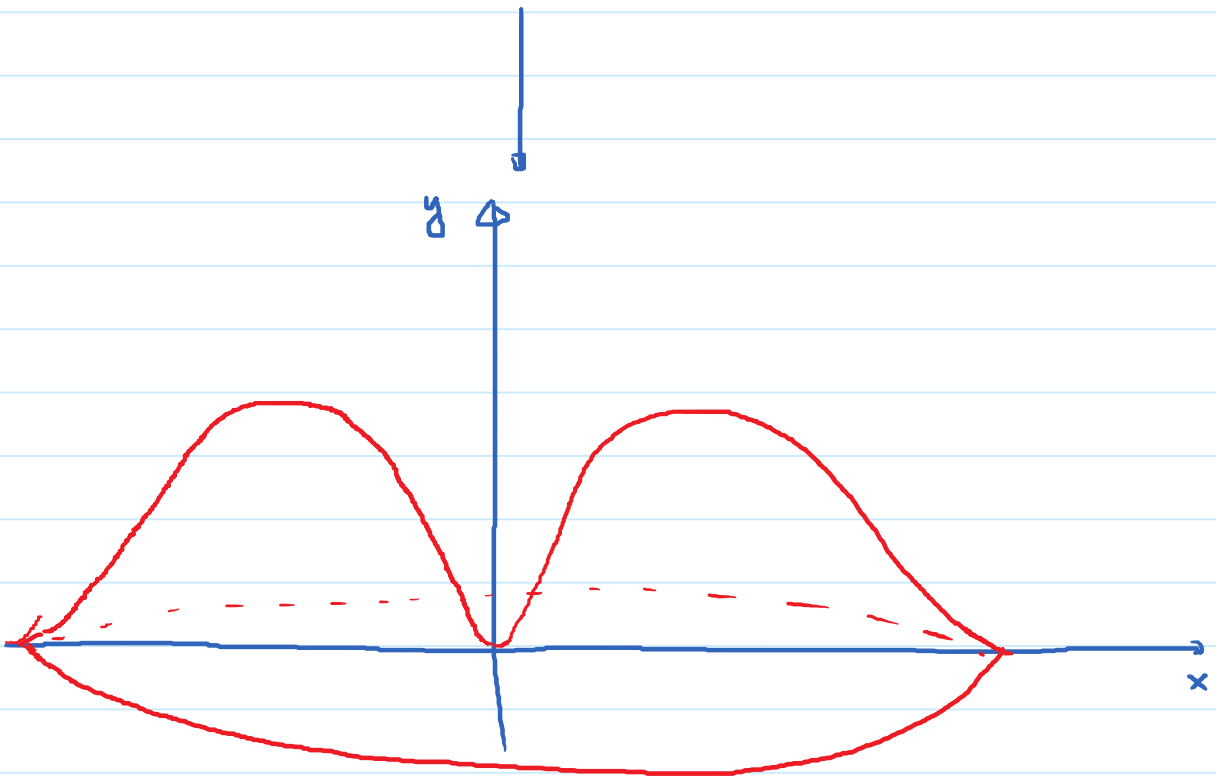
Volume - The Shell Method.

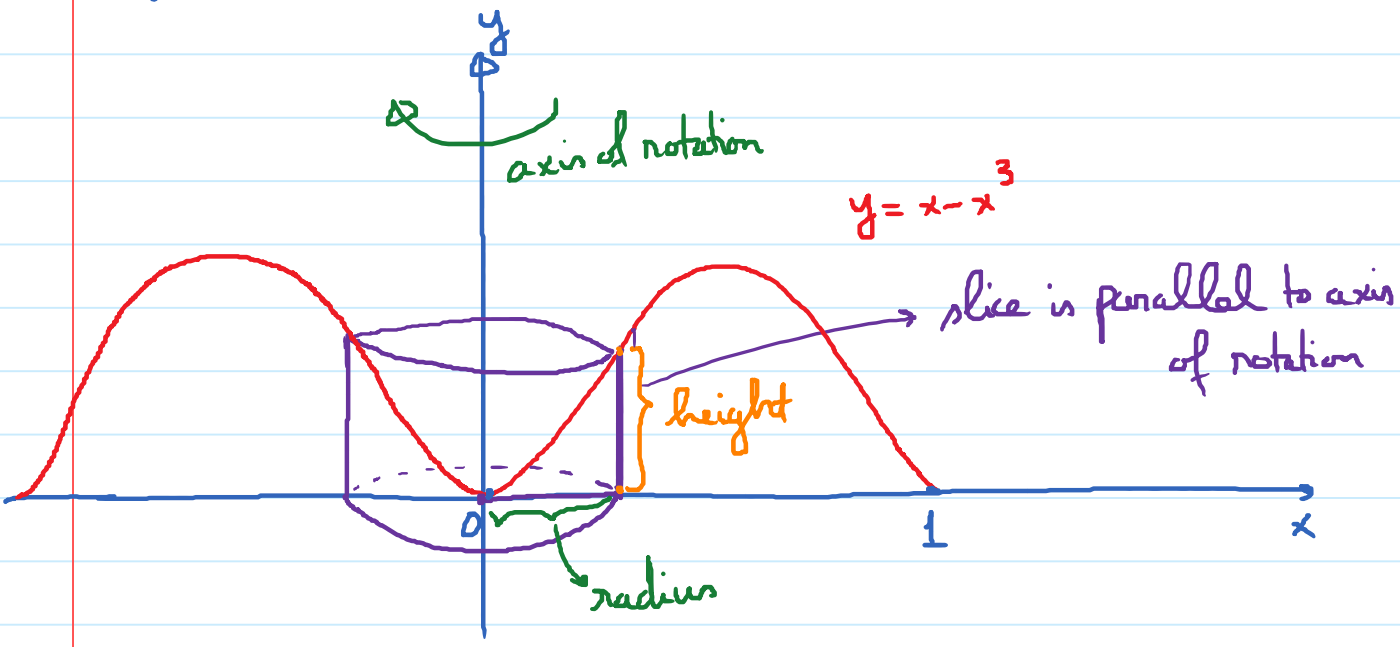
Tuesday, June 4, 2019 2:11 PM

E.g.





→ Shell Method



$$\text{cross-section area} = (\text{circumference})\cdot(\text{height})$$
$$= 2\pi\cdot(\text{radius})\cdot(\text{height})$$
$$= 2\pi\cdot(x)\cdot(x - x^3)$$

$$\text{Volume of a small shell} = 2\pi\cdot x\cdot(x - x^3)\cdot \underbrace{dx}_{\text{thickness}}$$

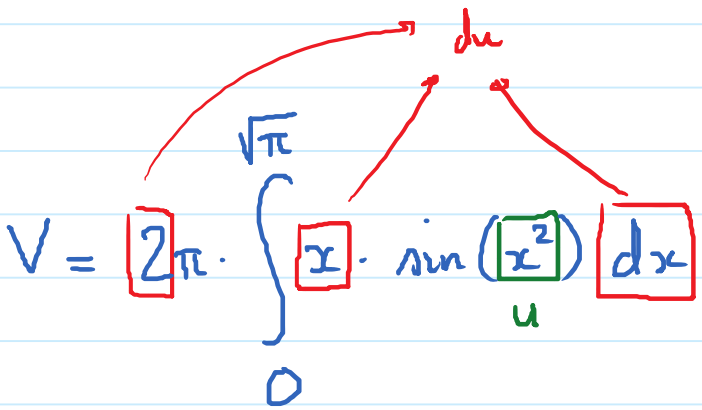
$$V_{\text{solid}} = \int_0^1 2\pi\cdot x\cdot(x - x^3)\cdot dx$$

$$= 2\pi\cdot\int_0^1 x(x - x^3)\,dx = 2\pi\int_0^1 (x^2 - x^4)\,dx$$

$$V = 2\pi \cdot \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_0^1$$

$$= 2\pi \left(\frac{1}{3} - \frac{1}{5} \right) = 2\pi \cdot \frac{2}{15} = \boxed{\frac{4\pi}{15}}.$$

E.g.2



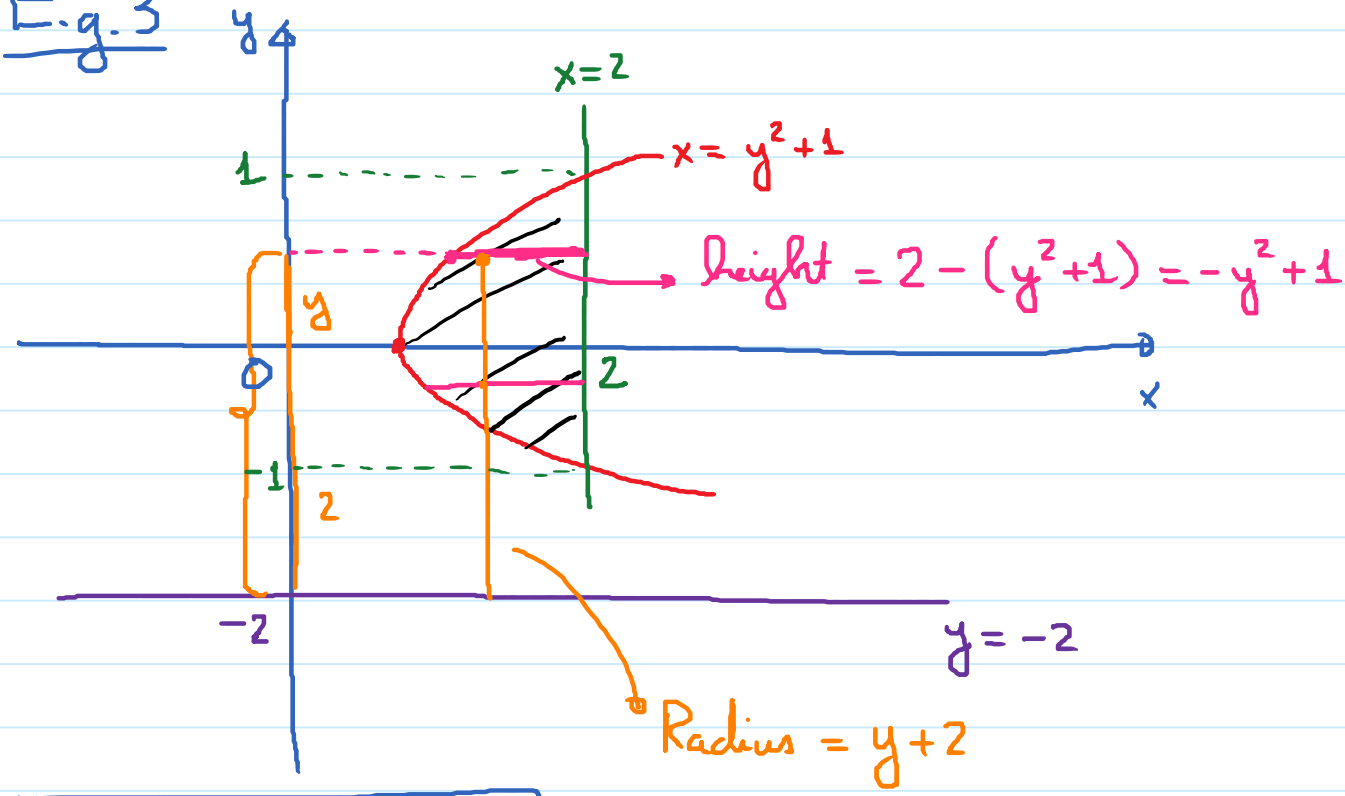
Let $u = x^2$. $du = 2x\,dx$

$$V = \pi \cdot \int_0^{\pi} \sin(u)\,du$$

$$= \pi \cdot \left(-\cos(u) \right) \Big|_0^{\pi}$$

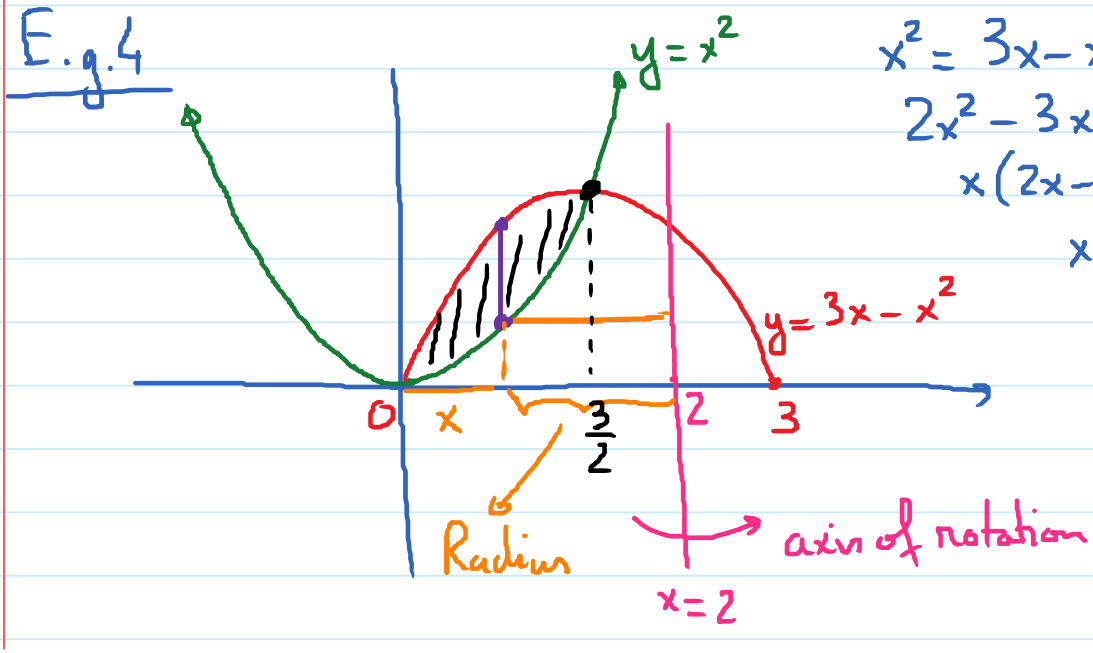
$$= \pi \cdot \left(-\cos(\pi) - \left(-\cos(0) \right) \right) = \boxed{2\pi}.$$

E.g. 3



$$V = 2\pi \cdot \int_{-1}^{1} (y+2)(-y^2+1)\,dy$$

E.g. 4



$$x^2 = 3x - x^2$$
$$2x^2 - 3x = 0$$
$$x(2x-3) = 0$$
$$x = 0 \; ; \; x = \frac{3}{2}$$

Height $= 3x - x^2 - x^2 = 3x - 2x^2$.

Radius $= 2 - x$.

$$V = 2\pi \cdot \int_0^{3/2} (2-x) \cdot (3x - 2x^2)\, dx$$