Volume - the Shell Method

Key formulas

To use the **shell method**, we "slice" a solid into cylindrical shells. The volume of a cylindrical shell is

$$V_{\text{shell}} = (\underbrace{2\pi \cdot \text{radius}}_{\text{circumference}})(\text{height})(\text{thickness}).$$

and then we integrate this expression to obtain the volume of the solid.
If the solid is obtained by revolve the region bounded the curve $y = f(x)$, $a \leq x \leq b$ and the x-axis about the y-axis, this formula translates to

$$V_{\text{solid}} = 2\pi \int_a^b \underbrace{x}_{\textbf{radius}} \underbrace{f(x)}_{\textbf{height}} \underbrace{dx}_{\textbf{thickness}} .$$

If the solid is obtained by revolve the region bounded the curve $x = f(y)$, $c \leq y \leq d$ and the y-axis about the x-axis, this formula translates to

$$V_{\text{solid}} = 2\pi \int_c^d \underbrace{y}_{\textbf{radius}} \underbrace{f(y)}_{\textbf{height}} \underbrace{dy}_{\textbf{thickness}} .$$

Example 1: Apply the shell method

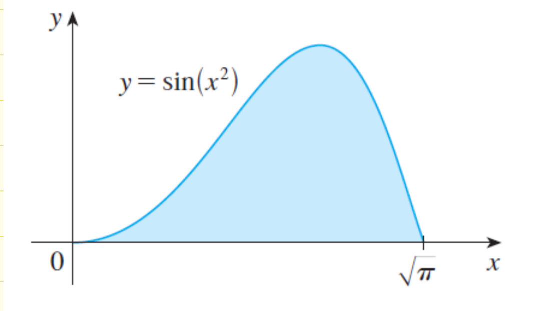
Find the volume of the solid formed by revolving the region bounded by $y = x - x^3$, $0 \leq x \leq 1$ and the x-axis about the y-axis.

Solution

Write the solution here

Example 2: Shell method is necessary

Find the volume of the solid formed by revolving the region bounded by $y = \sin(x^2)$, $0 \le x \le \sqrt{\pi}$ and the x-axis about the y-axis.



Solution

Write the solution here

Example 3: Region bounded by 2 curves revolved about a horizontal line (not x-axis)

Set up the integral (no need to evaluate) to find the volume of the solid formed by revolving the region bounded by $x = y^2 + 1$ and $x = 2$ about the line $y = -2$

Solution

Write the solution here

Example 4: Region bounded by 2 curves revolved about a vertical line (not y-axis)

Set up the integral (no need to evaluate) to find the volume of the solid formed by revolving the region bounded by $y = 3x - x^2$ and $y = x^2$ about the line $x = 2$.

Solution

Write the solution here