Arc length

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E.g 1. $y = 2x^{\frac{3}{2}} + 3$. over $[0, 8]$

$$\frac{dy}{dx} = 2 \cdot \frac{3}{2} \cdot x^{\frac{3}{2} - 1} = 3x^{\frac{1}{2}}$$

$$\text{Length} = \int_0^8 \sqrt{1 + \left(3x^{\frac{1}{2}}\right)^2}\, dx$$

$$= \int_0^8 \sqrt{\underbrace{1 + 9x}_{u}}\, dx \qquad \frac{du}{9}$$

Let $u = 1 + 9x$; $du = 9\,dx$

$$= \frac{1}{9} \int_1^{73} u^{\frac{1}{2}}\, du = \frac{1}{9} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big|_1^{73}$$

$$= \boxed{\frac{2}{27}\left((73)^{\frac{3}{2}} - 1\right)}$$

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$$\left(\ln(u)\right)' = \frac{1}{u} \cdot u'$$

E.g. 2

$$y = \ln\left(\boxed{\sec(x)}\right) \quad ; \quad \left[0, \frac{\pi}{4}\right]$$

$$\frac{dy}{dx} = \frac{1}{\cancel{\sec(x)}} \cdot \left(\cancel{\sec(x)}\tan(x)\right) = \tan(x)$$

$$L = \int_0^{\pi/4} \sqrt{1 + \tan^2(x)}\, dx$$

trig identity

$$= \int_0^{\pi/4} \sqrt{\sec^2(x)}\, dx = \int_0^{\pi/4} \sec(x)\, dx.$$

$$= \ln\left|\sec(x) + \tan(x)\right| \Big|_0^{\frac{\pi}{4}}$$

$$= \ln\left|\sec\left(\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}\right)\right| - \ln\left|\sec(0) + \tan(0)\right|$$

$$= \ln\left|\sqrt{2} + 1\right| - \ln|1| = \boxed{\ln\left|\sqrt{2} + 1\right|}$$

E.g. 3

$$y = kx^2$$

$$\frac{dy}{dx} = 2kx$$

$$L = 2 \cdot \int_0^w \sqrt{1 + (2kx)^2}\, dx$$

$$= 2 \cdot \int_0^w \sqrt{1 + 4k^2 x^2}\, dx$$

(w, h) is on parabola, so: $h = kw^2$.

So, $k = \frac{h}{w^2}$. Plug in, we get

$$L = 2 \cdot \int_0^w \sqrt{1 + 4\frac{h^2}{w^4} x^2}\, dx$$