Surface Area.

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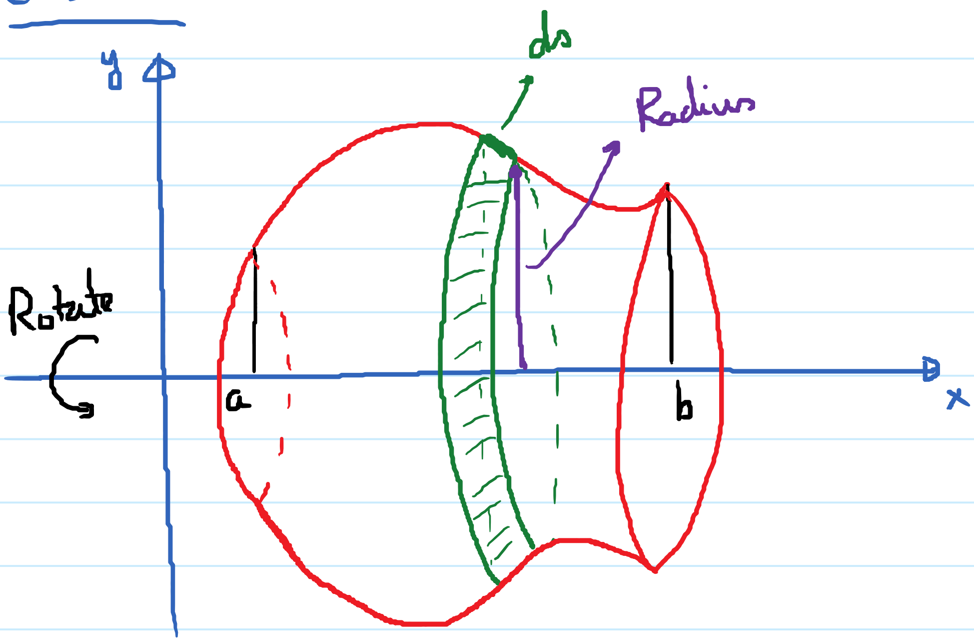
* Revolve $y = f(x)$; $a \le x \le b$.

Case 1: About x-axis



Area of band = (Circumference) · (height)

$$= 2\pi \cdot \text{radius} \cdot ds$$

$$= 2\pi \cdot f(x) \cdot \sqrt{1 + (f'(x))^2}\, dx$$

Surface area

$$\boxed{S = 2\pi \int_a^b f(x) \cdot \sqrt{1 + (f'(x))^2}\, dx}$$

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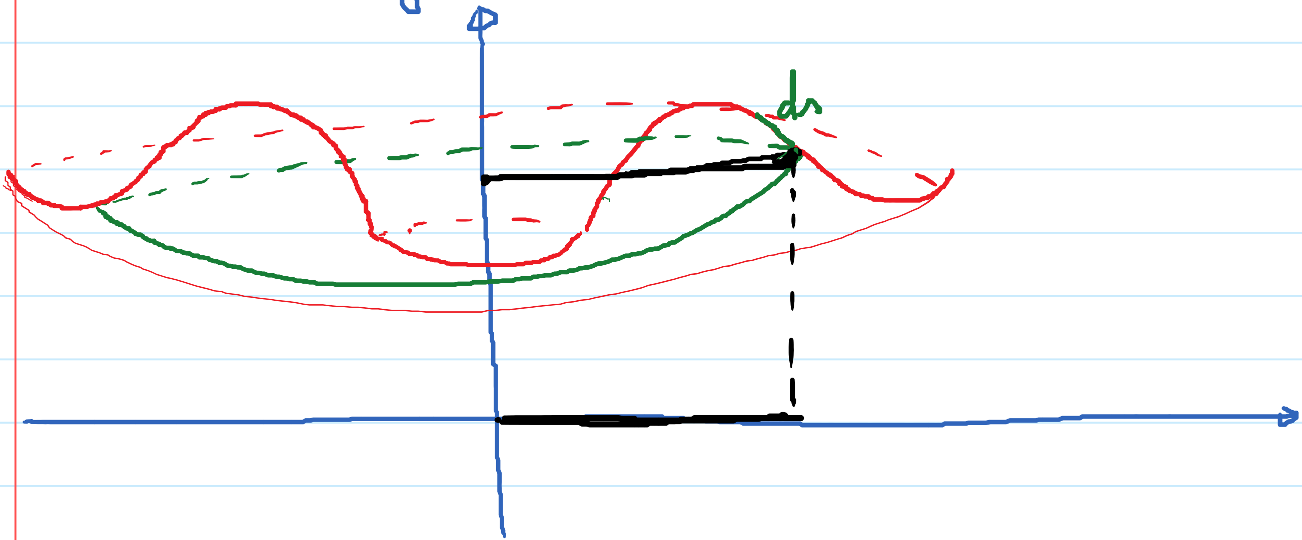
Case 2: about y-axis



area of band = $2\pi \cdot (\text{radius}) \cdot ds$

$$= 2\pi \cdot x \cdot \sqrt{1 + (f'(x))^2}\, dx$$

$$\text{Surface area} = 2\pi \cdot \int_a^b x \sqrt{1 + (f'(x))^2}\, dx.$$

$y = f(x)$; $a \leq x \leq b$.

Rotate about x-axis or y-axis.

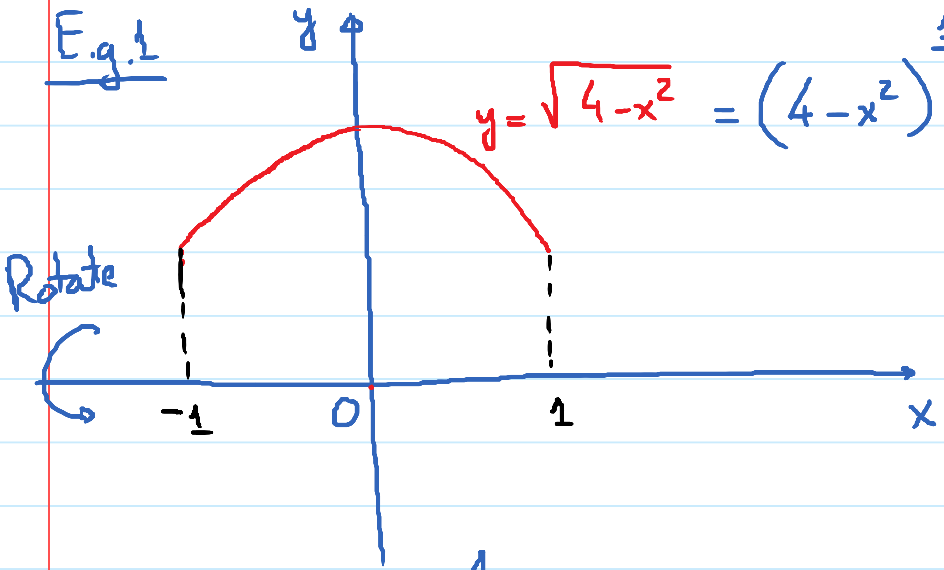
$$S = 2\pi \cdot \int_a^b (\text{Radius}) \cdot ds.$$

If rotate about x-axis : Radius $= f(x)$

If rotate about y-axis : Radius $= x$.

$ds = \sqrt{1 + (f'(x))^2}\, dx$ in both cases.

E.g.1



$= (4-x^2)^{\frac{1}{2}}$

$$S = 2\pi \cdot \int_{-1}^{1} (\text{radius}) \cdot ds$$

$$\text{radius} = \sqrt{4-x^2} \quad ; \quad ds = \sqrt{1 + (f'(x))^2}\, dx$$

$$\frac{dy}{dx} = \frac{1}{2}(4-x^2)^{-\frac{1}{2}} \cdot (-2x)$$

$$= -x(4-x^2)^{-\frac{1}{2}}$$

$$S = 2\pi \cdot \int_{-1}^{1} \sqrt{4-x^2} \cdot \sqrt{1 + \left(-x(4-x^2)^{-\frac{1}{2}}\right)^2}\, dx$$

$$= 2\pi \cdot \int_{-1}^{1} \sqrt{4-x^2} \cdot \sqrt{1 + x^2 \cdot (4-x^2)^{-1}}\, dx$$

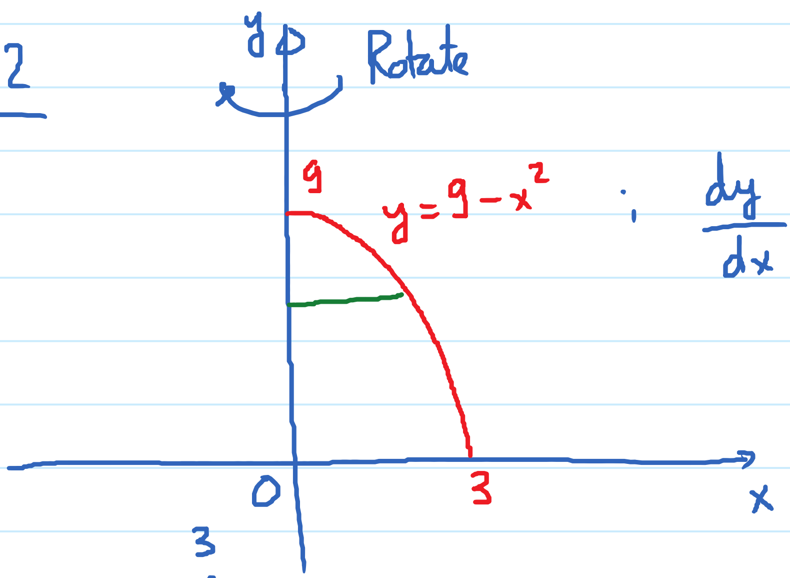
$$S = 2\pi \cdot \int_{-1}^{1} \sqrt{(4-x^2)\cdot\left(1 + x^2(4-x^2)^{-1}\right)}\,dx$$

$$= 2\pi \cdot \int_{-1}^{1} \sqrt{(4-x^2) + \boxed{(4-x^2)}x^2\boxed{(4-x^2)^{-1}}}\,dx$$

$$= 2\pi \cdot \int_{-1}^{1} \sqrt{4-x^2+x^2}\,dx$$

$$= 2\pi \cdot \int_{-1}^{1} 2\,dx = 2\pi \cdot 2x\Big|_{-1}^{1} = \boxed{8\pi}$$

E.g. 2



$; \quad \dfrac{dy}{dx} = -2x$

$$S = 2\pi \cdot \int_{0}^{3} (\text{radius}) \cdot ds \quad ; \quad \text{radius} = x$$

$$ds = \sqrt{1 + (f'(x))^2}\,dx$$

$$S = 2\pi \int_0^3 x \cdot \sqrt{1 + (-2x)^2}\, dx$$

$$= 2\pi \int_0^3 \cancel{x} \cdot \sqrt{\overset{u}{1 + 4x^2}}\, dx \quad \frac{du}{8\cancel{x}}$$

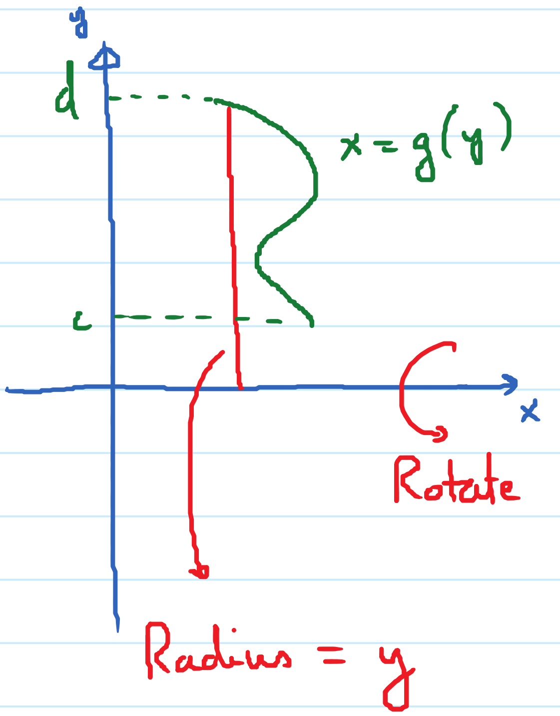
Let $u = 1 + 4x^2$. Then $du = 8x\,dx$

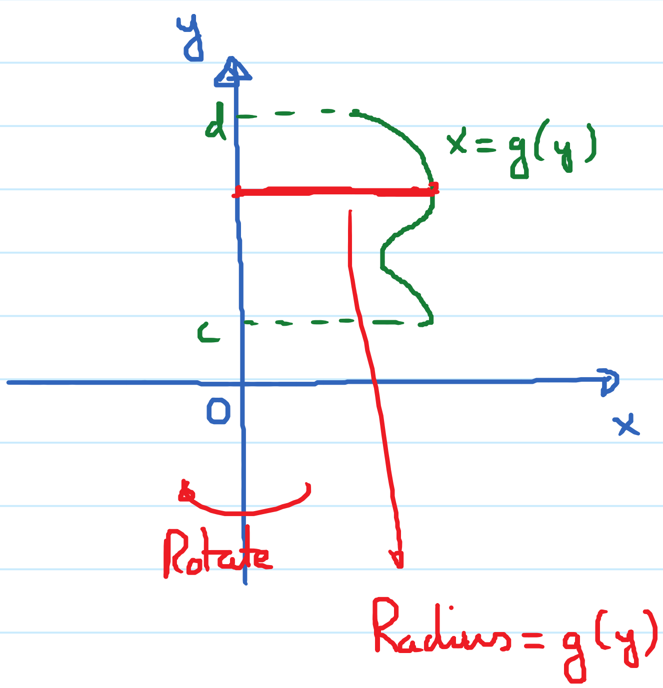
$$= \frac{\pi}{4} \int_1^{37} (u)^{\frac{1}{2}}\, du$$

$$= \frac{\pi}{4} \cdot \frac{2}{3}\, u^{\frac{3}{2}} \Big|_1^{37}$$

$$= \boxed{\frac{\pi}{6}\left((37)^{\frac{3}{2}} - 1\right)}$$

Revolve $x = g(y)$; $c \leq x \leq d$





$$S = 2\pi \int_c^d (\text{Radius})\, ds$$

In both cases: $ds = \sqrt{1 + (g'(y))^2}\, dy$.

About x-axis:

$$S = 2\pi \int_c^d y \cdot \sqrt{1 + (g'(y))^2}\, dy$$

About y-axis:

$$S = 2\pi \int_c^d g(y) \sqrt{1 + (g'(y))^2}\, dy.$$

E.g.3. $x = \ln(2y+1)$; $0 \leq y \leq 1$.

$$ds = \sqrt{1 + (g'(y))^2}\, dy.$$

$$\frac{dx}{dy} = \frac{2}{2y+1}$$

$$ds = \sqrt{1 + \left(\frac{2}{2y+1}\right)^2}\, dy.$$

About x-axis : Radius = y

$$S = 2\pi \int_0^1 y \sqrt{1 + \left(\frac{2}{2y+1}\right)^2}\, dy.$$

About y-axis : Radius = $g(y) = \ln(2y+1)$

$$S = 2\pi \int_0^1 \ln(2y+1) \cdot \sqrt{1 + \left(\frac{2}{2y+1}\right)^2}\, dy.$$