Area of a surface of revolution

Key formulas

The area S of the surface of revolution formed by revolving a smooth curve $y = f(x)$, $a \leq x \leq b$ about a horizontal or vertical axis is

$$S = 2\pi \int_a^b r(x) ds = 2\pi \int_a^b r(x) \sqrt{1 + [f'(x)]^2} dx$$

where $r(x)$ is the distance between the graph of f and the axis of revolution. We have: $r(x) = f(x)$ if we revolve f about the x-axis and $r(x) = x$ if we revolve f about the y-axis.
The area S of the surface of revolution formed by revolving a smooth curve $x = g(y)$, $c \leq y \leq d$ about a horizontal or vertical axis is

$$S = 2\pi \int_c^d r(y) ds = 2\pi \int_c^d r(y) \sqrt{1 + [g'(y)]^2} dy$$

where $r(y)$ is the distance between the graph of g and the axis of revolution. Hence, $r(y) = g(y)$ if we revolve f about the y-axis and $r(y) = y$ if we revolve g about the x-axis.

Example 1: Find area of a surface of revolution

Find the area of the surface formed by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$ about the x-axis.

Solution

Write the solution here

Example 2: Find area of a surface of revolution

Find the area of the surface formed by revolving the curve $y = 9 - x^2$, $0 \leq x \leq 3$ about the y-axis.

Solution

Write the solution here

Example 3: Find area of a surface of revolution

Set up an integral (no need to evaluate) to find the area of the surface formed by revolving the curve $x = \ln(2y+1)$, $0 \leq y \leq 1$ about (a) the x-axis and (b) the y-axis.

Solution

Write the solution here