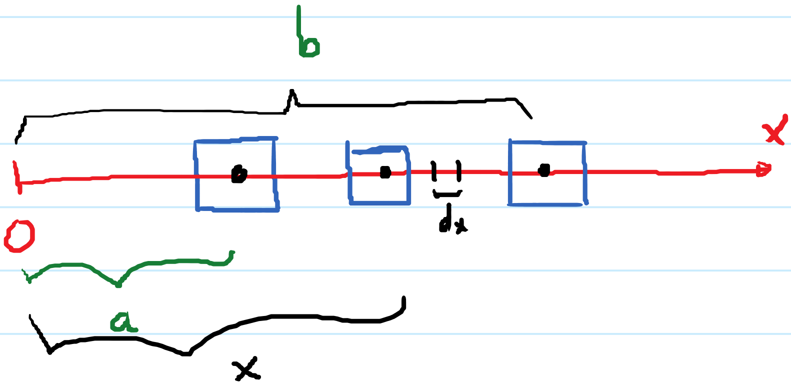
Work

Thursday, June 6, 2019 11:59 AM



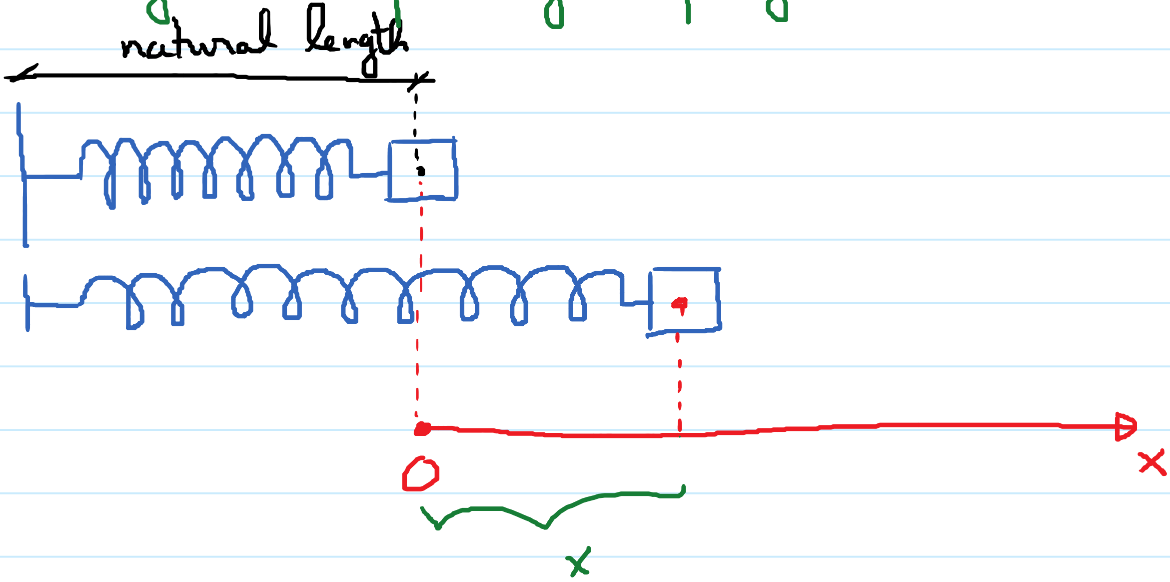
Varying force

Force acting on the object at location x is $\boxed{F(x)}$

Work done by F in moving object from $x = a$ to $x = b$ is

$$\boxed{W = \int_a^b F(x)\,dx}$$

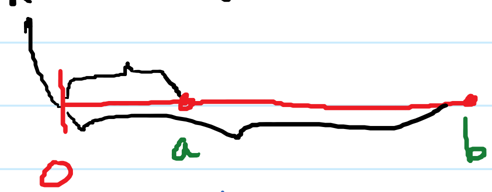
Stretching or Compressing a spring



Hooke's Law : $\boxed{F(x) = kx}$; k: spring constant

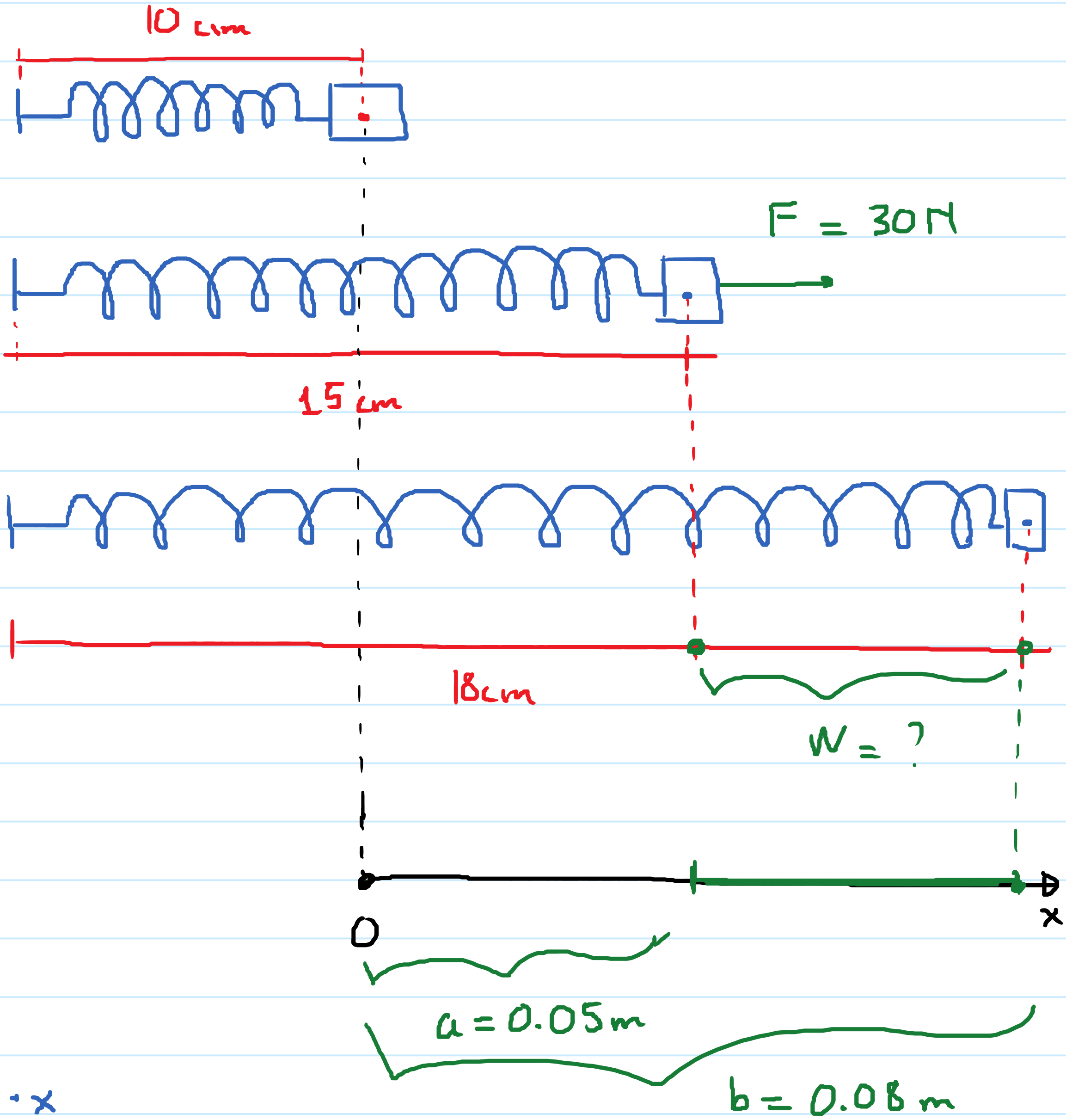
Thursday, June 6, 2019 12:09 PM

natural length



$$W = \int_a^b kx\,dx$$

E.g. 1:



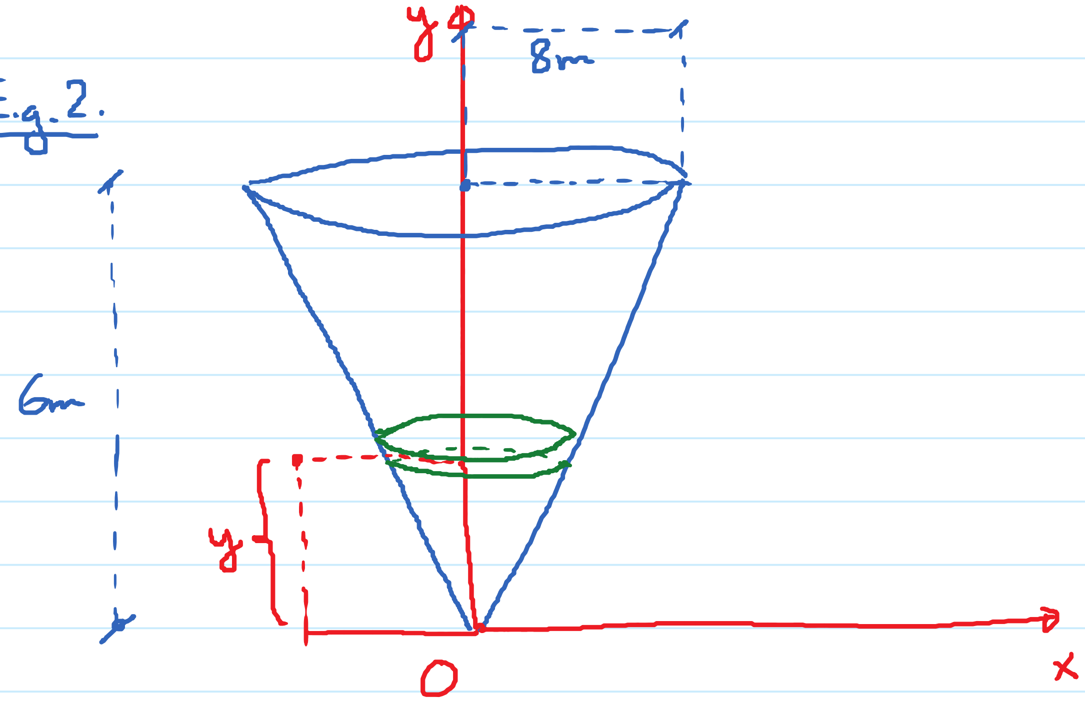
$$F = k \cdot x$$

$$\longrightarrow 30 = k \cdot (0.05) \rightarrow k = \frac{30}{0.05} = 600$$

$$W = \int_{0.05}^{0.08} F(x)\,dx = \int_{0.05}^{0.08} 600x\,dx$$

$$= 600 \cdot \frac{x^2}{2}\Bigg|_{0.05}^{0.08} = 300 \cdot \left((0.08)^2 - (0.05)^2\right) =$$

E.g. 2.



Step 1: Divide the body of water into small slices.

Find the expression for the work required to pump a slice out of the tank

Distance that slice needs to travel: $D(y) = 6 - y$.

Thursday, June 6, 2019 12:25 PM

$$\text{Force}_{slice} = m_{slice} \cdot \boxed{g}$$

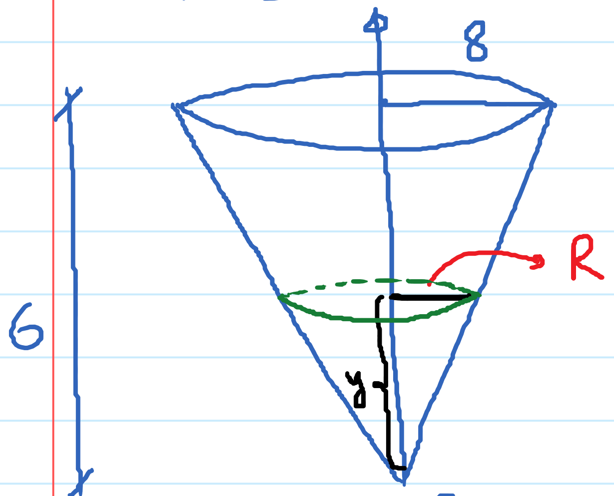
gravity

gravity constant = 9.8 m/s²

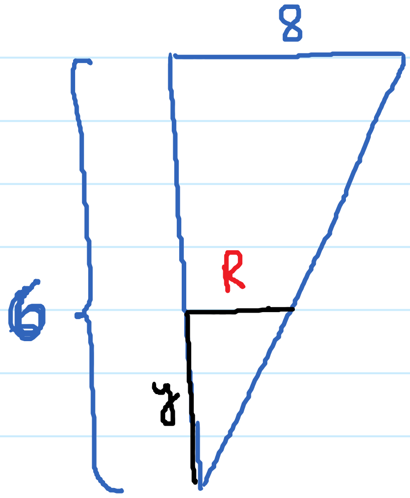
→ Find m_{slice}.

$$m_{slice} = V_{slice} \cdot \text{density} = V_{slice} \cdot 1000$$

$$V_{slice} = (\text{Base area}) \cdot (dy)$$



Base area of slice $= \pi \cdot (\text{Radius})^2$



$$\frac{R}{y} = \frac{8}{6} \rightarrow \boxed{R = \frac{4}{3} y}$$

Radius of a slice at height y.

$$F_{slice} = \overbrace{\underbrace{\underbrace{\pi \cdot \left(\frac{4}{3}y\right)^2}_{\text{Base area}} \cdot \underbrace{dy}_{\text{thickness}}}_{\text{volume}} \cdot \underbrace{1000}_{\text{density}}}^{m_{slice}} \cdot \overbrace{(9.8)}^{g}$$

$$F_{slice} = 9800\pi \cdot \frac{16}{3} y^2 \, dy.$$

$$W_{slice} = \underbrace{\left[\left(9800\pi \cdot \frac{16}{3}\right) y^2 \, dy\right]}_{F_{slice}} \cdot \underbrace{(6-y)}_{D_{slice}}$$

$$W_{slice} = 9800\pi \cdot \frac{16}{3} \, y^2 (6-y) \, dy$$

Total Work in pumping the entire body of water out of this tank:

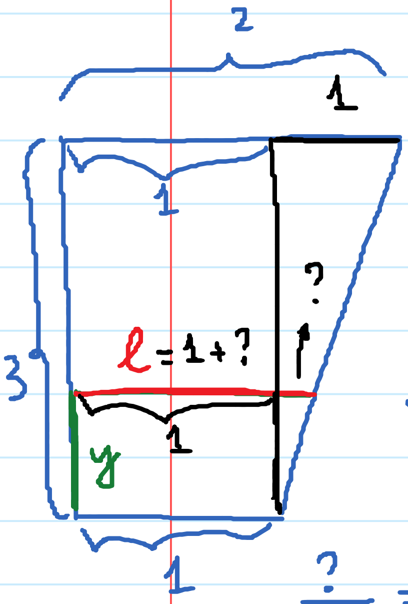
$$W = \int_0^6 \left(9800\pi \cdot \frac{16}{3}\right) y^2 (6-y) \, dy$$

$$= 9800\pi \cdot \frac{16}{3} \int_0^6 (6y^2 - y^3)\, dy$$

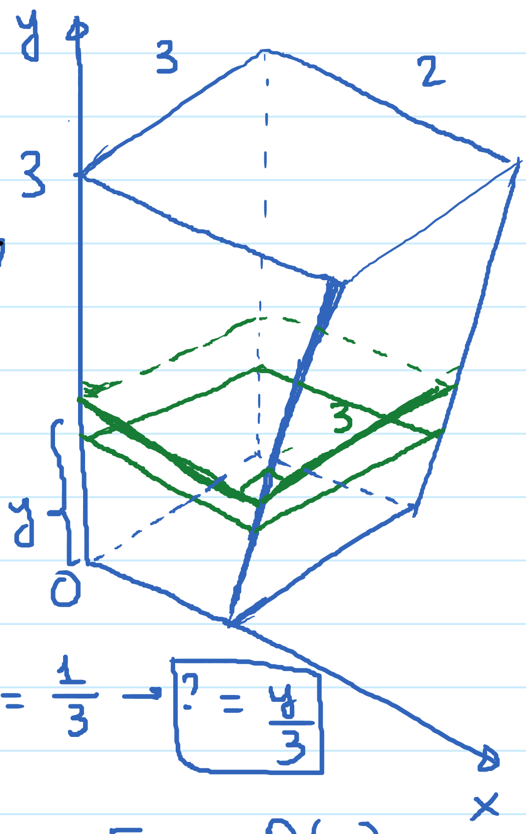
$$= 9800\pi \cdot \frac{16}{3} \left(2y^3 - \frac{y^4}{4} \Big|_0^6 \right)$$

$$= 9800\pi \cdot \frac{16}{3} \left(2(6)^3 - \frac{(6)^4}{4} \right)$$

Eg.3.



$$\frac{?}{y} = \frac{1}{3} \rightarrow \boxed{? = \frac{y}{3}}$$



$$m_{slice} = (\text{base area})\, dy.$$

Base → rectangle

$$\text{Base area} = \text{length} \cdot \text{width}$$

$$= l \cdot 3$$

$$= \left(1 + \frac{y}{3}\right) \cdot 3$$

$$W_{slice} = F_{slice} \cdot D(y) = F_{slice} \cdot (\cancel{3}\,6 - y)$$

$$F_{slice} = m_{slice} \cdot \text{density} = m_{slice}(53.1)$$

$$= \left(1 + \frac{y}{3}\right) \cdot 3 \cdot (53.1)$$

Thursday, June 6, 2019 12:52 PM

$$W_{slice} = \left(1 + \frac{y}{3}\right)\left(\overset{6}{\cancel{3}} - y\right) \cdot 3 \cdot (53.1)\, dy$$

$$W = \int_0^3 \left(1 + \frac{y}{3}\right)\left(\overset{6}{\cancel{3}} - y\right) \cdot 3 \cdot (53.1)\, dy$$

$$= 159.3 \int_0^3 \left(\overset{6}{\cancel{3}} - y + y - \frac{y^2}{3}\right) dy$$

$$= 159.3 \left(\overset{6}{\cancel{3}}y - \frac{y^3}{9}\right)\Bigg|_0^3$$

$$= 159.3\,(18 - 3)$$

$$= 2389.5 \text{ (J)}$$