Work

Key formulas

If a varying force $F(x)$ moves an object from $x = a$ to $x = b$, then the work done by the force is

$$W = \int_a^b F(x)dx.$$

Spring problem: The force is given by Hooke's law $F(x) = kx$ where k is the spring constant, x is the distance that the spring is compressed or stretched from its natural length.
Pumping liquid out of a tank problem: We divide the body of liquid into thin layers. The force to move a layer must overcome the force of gravity which is given by $F(y) = m(y)g$ where $m(y)$ is the mass (**in kg**) of a thin layer of liquid or $F(y) = w(y)$ where $w(y)$ is the weight (**in pounds**) of a thin layer of liquid. The mass $m(y)$ or the weight $w(y)$ is given by

$$\text{density} \cdot \text{volume} = \underbrace{\delta}_{\text{density}} \cdot \underbrace{A(y)}_{\text{area}} \cdot \underbrace{dy}_{\text{thickness}} .$$

The work to move a thin layer is $F(y)D(y)$ where $D(y)$ is the distance that the layer must be moved. To find the total work done in emptying the entire tank we find the integral $\int_a^b F(y)D(y)$ where a and b are the ranges of y that corresponds to the body of liquid.

Example 1: Stretching a spring

A force of 30N stretches a spring from its natural length of 10cm to a length of 15cm. How much work is done in stretching the spring from 15cm to 18cm.

Solution

Write the solution here

Example 2: Pumping water

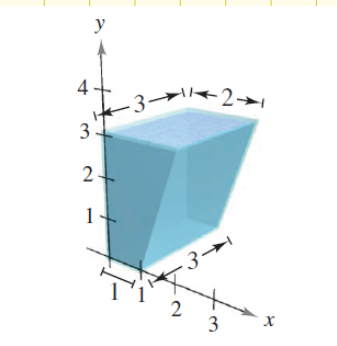
An open tank has the shape of a inverted circular cone. The base of the tank has a diameter of 8m and it is 6m high. Find the work done in emptying the tank by pumping the water over the top of the tank. Density of water is 1000kg/m^3.

Solution

Write the solution here

Example 3: Pumping fuel

The fuel tank on a truck has the shape as in the figure. The engine is 3ft above the top of the fuel tank. Find the work done by the fuel pump in raising a full tank of fuel to the level of the engine. Density of fuel is 53.1 pounds per cubic foot.



Solution

Write the solution here