Integration by Parts

Thursday, June 6, 2019 2:12 PM

Product Rule:

$$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + f'(x)g(x)$$

$$\longrightarrow \frac{d}{dx}\left[f(x)g(x)\right] - f'(x)g(x) = f(x)g'(x)$$

$$\int f(x)g'(x)\,dx = \cancel{\int \frac{d}{dx}}\left[f(x)g(x)\right] - \int f'(x)g(x)\,dx$$

$$\int f(x)\underbrace{\boxed{g'(x)dx}}_{dv} = f(x)g(x) - \int \boxed{f'(x)}\,g(x)\,\boxed{dx}$$

du

Let $u = f(x)$; $v = g(x)$. $dv = g'(x)dx$

$du = f'(x)dx$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

E.g 1. 1

$$\int \underbrace{x}_{u} \underbrace{\sin x \, dx}_{dv} = \boxed{u}\boxed{v} - \int \boxed{v}\boxed{du}$$

$u \to x$, $v \to -\cos x$, $v \to -\cos x$, $du \to dx$

$$\begin{cases} u = x \\ dv = \sin x \, dx \end{cases} \longrightarrow \begin{cases} du = dx \\ v = \int \sin x \, dx = -\cos x \end{cases}$$

$$= x \cdot (-\cos x) - \int (-\cos x) \, dx$$

$$\int x \sin x \, dx = -x\cos x + \int \cos x \, dx$$

$$\boxed{\int x \sin x \, dx = -x\cos x + \sin x + C}$$

2. $\int \boxed{x^5}\boxed{\ln(x)} \, dx$

$x^5 \to A$, $\ln(x) \to L$

$$\begin{cases} u = \ln(x) \\ dv = x^5 \, dx \end{cases} \longrightarrow \begin{cases} du = \frac{1}{x} \, dx \\ V = \frac{x^6}{6} \end{cases}$$

$$\int x^5 \ln(x)\,dx = \underbrace{(\ln(x))}_{u} \cdot \underbrace{\frac{x^6}{6}}_{v} - \int \underbrace{\frac{x^6}{6}}_{v} \cdot \underbrace{\frac{1}{x}\,dx}_{du}$$

$$= \frac{x^6 \ln(x)}{6} - \frac{1}{6}\int x^5\,dx$$

$$= \boxed{\frac{x^6 \ln(x)}{6} - \frac{x^6}{36} + C}$$

E.g.2

1 $\int \ln x\,dx$

$$\begin{cases} u = \ln x \\ dv = dx \end{cases} \longrightarrow \begin{cases} du = \frac{1}{x}\,dx \\ v = x \end{cases}$$

$$\int \ln x\,dx = x\ln x - \int x \cdot \frac{1}{x}\,dx$$

$$= x\ln x - \int 1\,dx = \boxed{x\ln x - x + C}$$

For definite integrals

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

2. $$\int_0^1 \arcsin(x)\,dx$$

$$\begin{cases} u = \arcsin(x) \\ dv = dx \end{cases} \longrightarrow \begin{cases} du = \dfrac{1}{\sqrt{1-x^2}}\,dx \\ v = x \end{cases}$$

$$\int_0^1 \arcsin(x)\,dx = x\arcsin(x)\Big|_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= \frac{\pi}{2} - \boxed{\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx} \longrightarrow 1$$

(found below)

$$= \boxed{\frac{\pi}{2} - 1}$$

Thursday, June 6, 2019 2:37 PM

$$\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx = \int_0^1 \cancel{x}\left(\overset{w}{\boxed{1-x^2}}\right)^{-\frac{1}{2}} dx \rightarrow \frac{dw}{-2\cancel{x}}$$

$$w = 1-x^2 \;;\; dw = -2x\,dx \;;\; dx = \frac{dw}{-2x}$$

$$-\frac{1}{2}\int_1^0 (w)^{-\frac{1}{2}}\,dw = \frac{1}{2}\int_0^1 (w)^{-\frac{1}{2}}\,dw$$

$$= \frac{1}{2}\cdot 2\cdot w^{1/2}\Big|_0^1 = 1$$

E.g. 3 $\int x^2 e^x \, dx$

$$\begin{cases} u = x^2 \\ dv = e^x \, dx \end{cases} \longrightarrow \begin{cases} du = 2x \, dx \\ v = e^x \end{cases}$$

$$\int x^2 e^x \, dx = x^2 e^x - 2 \boxed{\int x e^x \, dx}$$

$$\int x e^x \, dx = x e^x - \int e^x \, dx = \boxed{x e^x - e^x + C}$$

$$\begin{cases} u = x \\ dv = e^x \, dx \end{cases} \longrightarrow \begin{cases} du = dx \\ v = e^x \end{cases}$$

$$\int x^2 e^x \, dx = x^2 e^x - 2(x e^x - e^x) + C$$

6/6/2019 2:51 PM

$$= \boxed{x^2 e^x - 2x e^x + 2e^x + C}$$

2 $\int e^x \sin(x)\,dx$

$$\begin{cases} u = \sin(x) \\ dv = e^x\,dx \end{cases} \longrightarrow \begin{cases} du = \cos(x)\,dx \\ v = e^x \end{cases}$$

$$\int e^x \sin(x)\,dx = e^x \sin x - \boxed{\int e^x \cos(x)\,dx}$$

$$\begin{cases} u = \cos(x) \\ dv = e^x\,dx \end{cases} \longrightarrow \begin{cases} du = -\sin(x)\,dx \\ v = e^x \end{cases}$$

$$\int e^x \cos(x)\,dx = e^x \cos(x) - \int e^x(-\sin x)\,dx$$

$$= \boxed{e^x \cos(x) + \int e^x \sin(x)\,dx}$$

$$\int e^x \sin(x)\,dx = e^x \sin(x) - \left[e^x \cos(x) + \int e^x \sin(x)\,dx\right]$$

$$\int e^x \sin(x)\,dx = e^x \sin(x) - e^x \cos(x) - \underbrace{\int e^x \sin(x)\,dx}$$

Thursday, June 6, 2019 2:55 PM

$$2\int e^x \sin(x)\,dx = e^x \sin(x) - e^x \cos(x) + C$$

$$\int e^x \sin(x)\,dx = \boxed{\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x) + C}$$