$$2\int e^x \sin(x)\,dx = e^x \sin(x) - e^x \cos(x) + C$$

$$\int e^x \sin(x)\,dx = \boxed{\frac{1}{2}e^x \sin(x) - \frac{1}{2}e^x \cos(x) + C}$$

Tabular method for integration by parts

E.g. 4. $\int x^3 \cos(2x)\,dx$

Tabular method.

diff.	int.	
$u = x^3$	$\cos(2x) = dv$	
$3x^2$	$\frac{1}{2}\sin(2x)$	$(+)$
$6x$	$-\frac{1}{4}\cos(2x)$	$(-)$
6	$-\frac{1}{8}\sin(2x)$	$(+)$
0	$\frac{1}{16}\cos(2x)$	$(-)$

$$\int \cos(kx)\,dx = \frac{1}{k}\sin(kx)$$

$$\int \sin(kx)\,dx = -\frac{1}{k}\cos(kx)$$

Ans: $\int x^3\cos(2x)\,dx = +\frac{1}{2}x^3\sin(2x) - \left(-\frac{3}{4}x^2\cos(2x)\right) + \left(-\frac{3}{4}x\sin(2x)\right) - \left(\frac{3}{8}\cos(2x)\right)$

$$\int x^3\cos(2x)\,dx = \boxed{\frac{1}{2}x^3\sin(2x) + \frac{3}{4}x^2\cos(2x) - \frac{3}{4}x\sin(2x) - \frac{3}{8}\cos(2x) + C}$$

Reduction formula:

$$\int \sin^n x\,dx = -\frac{1}{n}\cos(x)\sin^{n-1}(x) + \frac{n-1}{n}\int \sin^{n-2}(x)\,dx$$

for $n \geq 2$.

E.g. 5.

1. $$\int \sin^2(x)\,dx = -\frac{1}{2}\cos(x)\sin(x) + \frac{1}{2}\int 1\,dx$$

$$= \boxed{-\frac{1}{2}\cos(x)\sin(x) + \frac{x}{2} + C}$$

2. $$\int \sin^4(x)\,dx = -\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\boxed{\int \sin^2(x)\,dx}$$

$$= -\frac{1}{4}\cos(x)\sin^3(x) + \frac{3}{4}\left(-\frac{1}{2}\cos(x)\sin(x) + \frac{x}{2}\right) + C$$

Walli's formula.

$$\int_0^{\pi/2} \cos^4(x)\,dx = \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{\pi}{2} = \frac{3\pi}{16}.$$

$$\int_0^{\pi/2} \sin^5(x)\,dx = \frac{2}{3}\cdot\frac{4}{5} = \frac{8}{15}.$$

$$* \int \sin^n(x)\,dx = -\frac{1}{n}\cos(x)\sin^{n-1}(x) + \frac{n-1}{n}\int \sin^{n-2}(x)\,dx$$

Derivation:

$$\int \underbrace{\sin^{n-1}(x)}_{u}\ \underbrace{\sin(x)\,dx}_{dv} = uv - \int v\,du$$

$$\begin{cases} u = \sin^{n-1}(x) \\ dv = \sin(x)\,dx \end{cases} \longrightarrow \begin{cases} du = (n-1)\sin^{n-2}(x)\cdot\cos(x) \\ v = -\cos(x) \end{cases}$$

$$\int \sin^n(x)\,dx = -\cos(x)\sin^{n-1}(x) + \int (n-1)\sin^{n-2}(x)\,\overset{(1-\sin^2 x)}{\cos^2(x)}\,dx$$

$$= -\cos(x)\sin^{n-1}(x) + (n-1)\underbrace{\int \left(\sin^{n-2}(x) - \sin^n(x)\right)dx}$$

Monday, June 10, 2019 12:31 PM

$$\int \sin^n(x)\,dx = -\cos(x)\sin^{n-1}(x) + (n-1)\int \sin^{n-2}(x)\,dx$$
$$- (n-1)\int \sin^n(x)\,dx$$

$$\boxed{\int \sin^n(x)\,dx} + (n-1)\boxed{\int \sin^n(x)\,dx} = -\cos(x)\sin^{n-1}(x)$$
$$+ (n-1)\int \sin^{(n-2)}(x)\,dx$$

$$n\int \sin^n(x)\,dx = -\cos(x)\sin^{n-1}(x) + (n-1)\int \sin^{(n-2)}(x)\,dx$$

$$\int \sin^n(x)\,dx = -\frac{1}{n}\cos(x)\sin^{n-1}(x) + \frac{n-1}{n}\int \sin^{(n-2)}(x)\,dx$$