Integration by parts

Key formulas

Recall the Product Rule from Cal 1

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x).$$

Take the integral of both sides and rearrange, we obtain the Integration by parts formula:

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx.$$

Equivalent formula: let $u = f(x)$, $v = g(x)$, then $du = f'(x)dx$ and $dv = g'(x)dx$, so the above formula becomes

$$\int udv = uv - \int vdu$$

The formula for definite integrals is

$$\int_a^b udv = uv\Big|_a^b - \int_a^b vdu.$$

A rule of thumb for choosing u is LIATE. This says whichever comes first in the following list should be chosen as u

- **L**: **L**ogarithmic functions such as $\ln(x)$, $\log(x)$, etc.
- **I**: **I**nverse trigonometric functions such as $\arcsin(x)$, $\arccos(x)$, $\arctan(x)$, etc.
- **A**: **A**lgebraic functions such as $x^2, 2x^3$, etc.
- **T**: **T**rigonometric functions such as $\sin(x), \cos(x), \tan(x)$, etc.
- **E**: **E**xponential functions such as $e^x, 2^x$, etc.

Reduction formulas for integrating powers of sine and cosine (these can be deduced using integration by parts)

$$\int \sin^n(x)dx = -\frac{1}{n}\cos(x)\sin^{n-1}(x) + \frac{n-1}{n}\int \sin^{n-2}(x)dx, \text{ for integer } n \geq 2.$$

$$\int \cos^n(x)dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)dx, \text{ for integer } n \geq 2.$$

These formulas give rise to Wallis'formulas

$$\int_0^{\pi/2} \cos^n(x)dx = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right)\cdots\left(\frac{n-1}{n}\right) \text{ if } n \geq 3 \text{ is odd.}$$

$$\int_0^{\pi/2} \cos^n(x)dx = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right)\cdots\left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right) \text{ if } n \geq 2 \text{ is even.}$$

The above formulas are also valid if $\cos^n(x)$ is replaced by $\sin^n(x)$.

Example 1: Apply the integration by parts formula

Find the given integral

1. $\int x \sin(x) dx$
2. $\int x^5 \ln(x) dx$

Solution

Write the solution here

Example 2: Integrand with a single term

Find the given integral

1. $\int \ln(x) dx$
2. $\int_0^1 \arcsin(x) dx$

Solution

Write the solution here

Example 3: Using integrations by parts repeatedly

Find the given integral

1. $\int x^2 e^x dx$
2. $\int e^x \sin(x) dx$

Solution

Write the solution here

Example 4: The Tabular method

Find $\int x^3 \cos(2x) dx$.

Solution

Write the solution here

Example 5: Use the reduction formula

Use the reduction formula to evaluate the given integral

1. $\int \sin^2(x)dx$
2. $\int \sin^4(x)dx$

Solution

Write the solution here

Example 6: Use Wallis'formulas

Use Wallis'formulas to evaluate the given integral

1. $\int_0^{\pi/2} \cos^4(x)dx$
2. $\int_0^{\pi/2} \sin^5(x)dx$

Solution

Write the solution here