Name:_____

Student ID:_____

Section:_____

Instructor:_____

Math 2414 (Calculus II)
Practice Exam 2

Instructions:

- Work on scratch paper will not be graded.
- Show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer.
- **Please write neatly. If I cannot read your handwriting, you will not receive credit.**
- Simplify your answers as much as possible. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

Show all work in the space provided. Full credit will be given only if all steps are shown justifying your answer. Please write neatly and carefully, if I cannot read your handwriting, you will receive NO credit.

1. (10 points) Find the integral (show all work.)

$$\int \tan^5(x)\sec^3(x)dx.$$

2. (10 points) Find the integral (show all work.)

$$\int_0^a x^2\sqrt{a^2-x^2}dx, \text{ where } a>0 \text{ is a constant.}$$

3. (10 points) Find the integral (show all work.)

$$\int \frac{\sin x}{\cos^2(x) - 3\cos(x)} dx.$$

4. (10 points) Find the area of the region and find the volume of the solid formed by revolving the region about the x-axis where the region is bounded by the graph of $y = e^{-x}$ over $[0, \infty)$, the x-axis and the y-axis.

5. (10 points) Explain why the integral is improper and evaluate it (show all work.)

(a) $\int_0^\infty e^{-x}\cos(x)dx$

(b) $\int_0^5 \frac{1}{25-x^2}dx$

6. (10 points)

(a) Find the limit of the sequence $a_n = \tan\left(\dfrac{2n\pi}{1+8n}\right)$

(b) Find the formula for the **nth** term of the sequence $\dfrac{1}{2}, -\dfrac{2}{5}, \dfrac{3}{10}, -\dfrac{4}{17}, \ldots$.

7. (10 points) Find all values of x for which the series converges. Then write the sum of the series as a function of x.

$$\sum_{n=0}^{\infty}(-4)^n(x-5)^n.$$

8. (10 points)

(a) Find the sum of the series $\sum_{n=0}^{\infty} \left[\left(\frac{2}{3} \right)^n - \frac{1}{(n+1)(n+2)} \right]$. (show all work.)

(b) Explain why the series $\sum_{n=1}^{\infty} \ln \left(\frac{n^2+1}{2n^2+1} \right)$ diverges.

9. (10 points) Find the integral (show all work.)

$$\int \frac{x^2}{x^4 - 2x^2 - 8} dx.$$

10. (10 points) Show that for any positive integers m and n

(a) $\displaystyle\int_0^{2\pi} \sin(mx)\cos(nx)dx = 0$

(b) $\displaystyle\int_0^{2\pi} \sin(mx)\sin(nx)dx = 0$

(c) $\displaystyle\int_0^{2\pi} \cos(mx)\cos(nx)dx = 0$

END OF EXAM