Name:_____

Student ID:_____

Section:_____

Instructor:_____

Math 2414 (Calculus II)
Practice Exam 3

Instructions:

- Work on scratch paper will not be graded.
- Show all your work in the space provided. Full credit will be given only if the necessary work is shown justifying your answer.
- **Please write neatly. If I cannot read your handwriting, you will not receive credit.**
- Simplify your answers as much as possible. Expressions such as $\ln(1)$, e^0, $\sin(\pi/2)$, etc. must be simplified for full credit.

Show all work in the space provided. Full credit will be given only if all steps are shown justifying your answer. Please write neatly and carefully, if I cannot read your handwriting, you will receive NO credit.

1. (10 points) Explain why the integral test can be applied to the series. Then apply the test to determine whether the series converges or diverges.

$$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}.$$

2. (10 points) Use the limit comparison test to determine whether the series converges or diverges

$$\sum_{n=1}^{\infty} \frac{2n^2 - 1}{3n^5 + 2n + 1}.$$

3. (10 points) Determine the number of terms required to approximate the sum of the series with an error of less than 0.001

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n^3-1}.$$

4. (10 points) The terms of the series $\sum a_n$ are defined recursively as follows. Determine whether the series converges or diverges.

$$a_1 = 1, a_{n+1} = \frac{2+\cos n}{\sqrt{n}}a_n.$$

5. (10 points) Find the 4th Taylor polynomial $T_4(x)$ centered at 2 for $f(x) = \ln x$. Use $T_4(x)$ to estimate $f(2.1)$.

6. (10 points) Find the radius of convergence and interval of convergence for the given series. Make sure to test convergence at the endpoints of the interval.

$$\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}$$

7. (10 points) Find a geometric power series centered at α for the given function. Determine the interval of convergence. $f(x) = \dfrac{5}{2x-3}$, $\alpha = -3$.

8. (10 points) Find the first four nonzero terms of the Maclaurin series for the function

$$h(x) = e^x \arctan x.$$

9. (10 points) Use a power series to approximate the integral with an error of less than 0.0001: $\int_0^1 \cos(x^2)dx$

10. (10 points) Use differentiation to find a power series centered at 0 for $f(x) = \dfrac{x}{(1-x)^2}$. Use the result to find the sum of the series $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$

END OF EXAM