Polar Coordinates

Key formulas

The **polar coordinate system** consists of the origin O and the positive part of the *x*-axis (called the **polar axis**). Every point P in the plane corresponds to an order paired of numbers (r, θ) (called the **polar coordinates**) where r is the distance from O to P and θ can be chosen to be the angle measured counterclockwise from the polar axis to the ray OP. (Note that by convention an angle measured clockwise is negative.) This representation of a point with polar coordinates is not unique. For example, (r, θ) and $(r, \theta + 2\pi)$ correspond to the same point. Also, by convention, (r, θ) and $(-r, \theta + \pi)$ correspond to the same point.

The relationship between polar coordinates (r, θ) and rectangular coordinates (x, y) of a point is given by the following equations

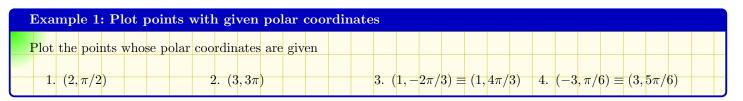
- Polar-to-Rectangular: $x = r \cos \theta$, $y = r \sin \theta$.
- Rectangular-to-Polar: $\theta = \arctan(\frac{y}{x})$ if the point is in QI or QIV, $\theta = \arctan(\frac{y}{x}) + \pi$ if the point is in QII or QIII. $r = \sqrt{x^2 + y^2}$.

Tangents to polar curves: The set of parametric equations for a polar curve $r = f(\theta)$ are

$$x = r\cos\theta = f(\theta)\cos\theta, \quad y = r\sin\theta = f(\theta)\sin\theta.$$

As a result,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}.$$



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Example 2: Polar-to-Rectangular	r Conversion
Plot the point whose polar coordinate	es are given and find the corresponding rectangular coordinates for the point.
1. $(2, 3\pi/4)$	$2. (-2, 5\pi/3)$

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Example 3: Rectar	ular-to-Polar Conversion	
Plot the point whose	ectangular coordinates are given and find the corresponding polar coordinates for the point.	
1. $(3\sqrt{3},3)$	$2. \ (-1, \sqrt{3}) \qquad \qquad 3. \ (2, -2) \qquad \qquad 4. \ (-5, 0)$	

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Example 4: Plot a polar curv	y hand	
Sketch the following curves:		
1. $r = 3$	2. $\theta = \frac{\pi}{3}$ 3. $r = \cos(2\theta)$	

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Example 5: Convert between polar	ar equations and rectangular equations
Convert the polar equation to a rectang	gular equation and identify the curve.
1. $r = 5\cos\theta$	2. $r = 2 \csc \theta$ 3. $r = \cot \theta \csc \theta$

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Example 6: Tangent lines to polar curves

- 1. Find dy/dx and the slope of the tangent line to the graph of $r = 2(1 \sin \theta)$ at the point where $\theta = 0$.
- 2. Find the points on the polar curve $r = 3\cos\theta$ where the tangent line is horizontal or vertical.

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