Polar Coordinates

Key formulas

The **polar coordinate system** consists of the origin O and the positive part of the x-axis (called the **polar axis**). Every point P in the plane corresponds to an order paired of numbers (r, θ) (called the **polar coordinates**) where r is the distance from O to P and θ can be chosen to be the angle measured counterclockwise from the polar axis to the ray OP. (Note that by convention an angle measured clockwise is negative.) This representation of a point with polar coordinates is not unique. For example, (r, θ) and $(r, \theta + 2\pi)$ correspond to the same point. Also, by convention, (r, θ) and $(-r, \theta + \pi)$ correspond to the same point.
The relationship between polar coordinates (r, θ) and rectangular coordinates (x, y) of a point is given by the following equations

- **Polar-to-Rectangular:** $x = r\cos\theta$, $y = r\sin\theta$.
- **Rectangular-to-Polar:** $\theta = \arctan(\frac{y}{x})$ if the point is in QI or QIV, $\theta = \arctan(\frac{y}{x}) + \pi$ if the point is in QII or QIII. $r = \sqrt{x^2 + y^2}$.

Tangents to polar curves: The set of parametric equations for a polar curve $r = f(\theta)$ are

$$x = r\cos\theta = f(\theta)\cos\theta, y = r\sin\theta = f(\theta)\sin\theta.$$

As a result,

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}.$$

Example 1: Plot points with given polar coordinates

Plot the points whose polar coordinates are given

1. $(2, \pi/2)$
2. $(3, 3\pi)$
3. $(1, -2\pi/3) \equiv (1, 4\pi/3)$
4. $(-3, \pi/6) \equiv (3, 5\pi/6)$

Solution

Write the solution here

Example 2: Polar-to-Rectangular Conversion

Plot the point whose polar coordinates are given and find the corresponding rectangular coordinates for the point.

1. $(2, 3\pi/4)$
2. $(-2, 5\pi/3)$

Solution

Write the solution here

Example 3: Rectangular-to-Polar Conversion

Plot the point whose rectangular coordinates are given and find the corresponding polar coordinates for the point.

1. $(3\sqrt{3}, 3)$
2. $(-1, \sqrt{3})$
3. $(2, -2)$
4. $(-5, 0)$

Solution

Write the solution here

Example 4: Plot a polar curve by hand

Sketch the following curves:

1. $r = 3$
2. $\theta = \frac{\pi}{3}$
3. $r = \cos(2\theta)$

Solution

Write the solution here

Example 5: Convert between polar equations and rectangular equations

Convert the polar equation to a rectangular equation and identify the curve.

1. $r = 5\cos\theta$
2. $r = 2\csc\theta$
3. $r = \cot\theta\csc\theta$

Solution

Write the solution here

Example 6: Tangent lines to polar curves

1. Find dy/dx and the slope of the tangent line to the graph of $r = 2(1 - \sin\theta)$ at the point where $\theta = 0$.
2. Find the points on the polar curve $r = 3\cos\theta$ where the tangent line is horizontal or vertical.

Solution

Write the solution here