

Area and Arc Length in Polar Coordinates

Key formulas

Area: The area of the region bounded by the polar curve $r = f(\theta)$ and the rays $\theta = a$ and $\theta = b$ is given by

$$A = \frac{1}{2} \int_a^b r^2 d\theta = \frac{1}{2} \int_a^b [f(\theta)]^2 d\theta.$$

The area of the region bounded by the polar curves $r = f(\theta)$, $r = g(\theta)$, the rays $\theta = a$ and $\theta = b$ where $f(\theta) \geq g(\theta) \geq 0$ is given by

$$A = \frac{1}{2} \int_a^b ([f(\theta)]^2 - [g(\theta)]^2) d\theta.$$

Arc Length: The length of a curve with polar equation $r = f(\theta)$, $a \leq \theta \leq b$ is given by

$$L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta.$$

Example 1: Find area

Find the area bounded by the curve $r = 3 \cos(3\theta)$ and the rays $\theta = -\pi/6$ and $\theta = \pi/6$

Solution

Write the solution here

Example 2: Find area

Find the area of the region that lies inside the curve $r = 3 \sin \theta$ and outside the curve $r = 1 + \sin \theta$.

Solution

Write the solution here

Example 3: Find arc length of a polar curve

Find the length of the polar curve over the given interval

1. $r = a, 0 \leq \theta \leq 2\pi$.

2. $r = 1 + \cos \theta, 0 \leq \theta \leq 2\pi$.

Solution

Write the solution here