Arc Length

Key formulas

The length of a smooth curve $y = f(x)$ from $x = a$ to $x = b$ is given by

$$L = \int_a^b ds = \int_a^b \sqrt{1 + \left[\frac{dy}{dx}\right]^2}\, dx = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx.$$

The length of a smooth curve $x = g(y)$ from $y = c$ to $y = d$ is given by

$$L = \int_c^d ds = \int_c^d \sqrt{1 + \left[\frac{dx}{dy}\right]^2}\, dy = \int_c^d \sqrt{1 + [g'(y)]^2}\, dy.$$

Example 1: Finding arc length

Find the arc length of the curve $y = 2x^{3/2} + 3$ over the interval $[0, 8]$.

Solution

Write the solution here

Example 2: Finding arc length

Find the arc length of the curve $y = \ln(\sec(x))$ over the interval $[0, \pi/4]$.

Solution

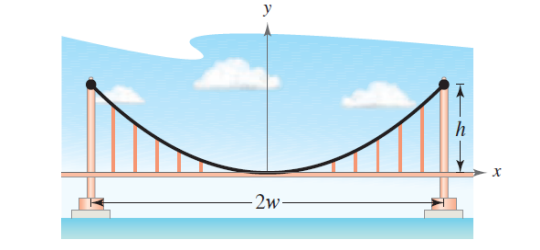
Write the solution here

Example 3: Application

The (parabolic) cable of a suspension bridge is modeled by the equation $y = kx^2$. Demonstrate that the length of the cable is given by the formula

$$L = 2\int_0^w \sqrt{1 + (\frac{4h^2}{w^4})x^2}dx.$$

where h and w are as in the figure.



Solution

Write the solution here