Due at the beginning of class on the day of Test 2

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2414, and the title of the worksheet.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Typer 1 improper integrals

Evaluate the given improper integral.

1. $\int_1^\infty \frac{3}{\sqrt[3]{x}}dx$
2. $\int_0^\infty x^2e^{-x}dx$
3. $\int_1^\infty \frac{\ln(x)}{x}dx$
4. $\int_{-\infty}^\infty \frac{x^2}{9+x^6}dx$

Problem 2: Typer 2 improper integrals

Explain why the integral is improper and evaluate it.

1. $\int_0^2 \frac{1}{\sqrt[3]{x-1}}dx$
2. $\int_0^1 x\ln(x)dx$
3. $\int_3^6 \frac{1}{\sqrt{36-x^2}}dx$
4. $\int_0^3 \frac{1}{x^2-6x+5}dx$

Problem 3: An application - Surface Area

Find the area of the surface formed by revolving the graph of $y = 2e^{-x}$ on the interval $[0, \infty)$ about the x-axis.

Problem 4: The Gamma Function

The Gamma Function $\Gamma(n)$ is defined as

$$\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx, \text{ for } n > 0.$$

1. Find $\Gamma(n)$ for $n = 1$ and $n = 2$.
2. Use integration by parts to show that $\Gamma(n+1) = n\Gamma(n)$. (Hint: let $u = x^{n-1}$ and $dv = e^{-x}dx$.)