Due at the beginning of class on the day of Test 3

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2414, and the title of the worksheet.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Using the integral test

Explain why the integral test can be applied to the series. Then apply the test to determine whether the series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{2}{3n+5}$
2. $\displaystyle\frac{1}{4}+\frac{2}{7}+\frac{3}{12}+\ldots+\frac{n}{n^2+3}+\ldots$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{(2n+3)^3}$
4. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$

Problem 2: Using the integral test

Explain why the integral test cannot be applied to the series to determine convergence or divergence

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{\cos^2(n)}{1+n^2}$

Problem 3: Using the p-series test

Determine whether the series converges or diverges

1. $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\sqrt{2}}}$
2. $\displaystyle 1+\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}+\frac{1}{4\sqrt{4}}+\frac{1}{5\sqrt{5}}+\ldots$

Problem 4: Remainder Estimate for the integral test (Optional Extra Credit Problem)

1. Suppose that the series $\displaystyle\sum_{n=1}^{\infty} a_n$ converges to the real number s. The **Nth Remainder** is the quantity $R_N = s - S_N$ where S_N is the sum of the first N terms of the series. If $a_n = f(n)$, then we have the following lower and upper bounds for the **Nth remainder**
$$\int_{N+1}^{\infty} f(x)dx \leq R_N \leq \int_{N}^{\infty} f(x)dx.$$
Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$ which converges by the p-series test. Use the above inequality to find N such that $R_N \leq 0.001$.
2. If we add S_N to both sides of the inequality in the previous part, we get
$$S_N + \int_{N+1}^{\infty} f(x)dx \leq s \leq S_N + \int_{N}^{\infty} f(x)dx.$$
This gives us lower bound and upper bound for the sum s of the series. Plug the value of N in the previous part in the above inequality to obtain estimates for the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$.