Due at the beginning of class on the day of Test 3

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2414, and the title of the worksheet.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Find radius of convergence and interval of convergence of a series

Find the radius of convergence and interval of convergence for the given series. Make sure to test convergence at the endpoints of the interval.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{x^{5n}}{n!}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{n! x^n}{(2n)!}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdot \cdots \cdot (2n)}$

Problem 2: Interval of convergece

Suppose that the series $\displaystyle\sum_{n=0}^{\infty} c_n 4^n$ converges, can you conclude that the following series also converge? Explain why.

1. $\displaystyle\sum_{n=0}^{\infty} c_n (-2)^n$
2. $\displaystyle\sum_{n=0}^{\infty} c_n (-4)^n$

Problem 3: Interval of convergence

Suppose that the series $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$, what can you conclude about the convergence or divergence of the following series? Explain why.

1. $\displaystyle\sum_{n=0}^{\infty} c_n$
2. $\displaystyle\sum_{n=0}^{\infty} c_n 8^n$
3. $\displaystyle\sum_{n=0}^{\infty} c_n (-3)^n$
4. $\displaystyle\sum_{n=0}^{\infty} (-1)^n c_n 9^n$

Problem 4: Term-by-term differentiation and integration

Find the series for $f'(x)$ and $\displaystyle\int f(x)dx$ and determine the interval of convergence for each series.

1. $f(x) = \displaystyle\sum_{n=0}^{\infty} \left(\frac{x}{3}\right)^n$
2. $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-1)^{n+1}}{n+1}$

Problem 5: Important power series

Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ and $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

1. Find the intervals of convergence of f and g.
2. Find the series for $f'(x)$ and show that $f'(x) = g(x)$.
3. Find the series for $g'(x)$ and show that $g'(x) = f(x)$.
4. Do you recognize the functions $f(x)$ and $g(x)$ as some familiar functions?