Worksheet 20: Representation of functions by power series

Due at the beginning of class on the day of Test 3

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2414, and the title of the worksheet.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Functions represented by geometric power series

Find a geometric power series centered at α for the given function. Determine the interval of convergence.

1. $f(x) = \dfrac{1}{3-x}$, $\alpha = 1$.
2. $f(x) = \dfrac{5}{2x-3}$, $\alpha = -3$.
3. $f(x) = \dfrac{3}{3x+4}$, $\alpha = 0$.

Problem 2: Find power series using basic operations

Find the partial fraction decomposition of the function and use it to find the power series for the function centered at 0. Determine the interval of convergence.

1. $g(x) = \dfrac{-2}{x^2-1}$.
2. $f(x) = \dfrac{4x}{x^2+2x-3}$.

Problem 3: Find power series by differentiation

1. Use the power series centered at 0 for $f(x) = \dfrac{1}{1+x}$ to find a power series centered at 0 for $g(x) = \frac{2}{(1+x)^3}$. (Hint: $g(x) = f''(x)$). Determine the interval of convergence of the series you found.
2. Use the power series centered at 0 for $f(x) = \dfrac{1}{1-x}$ to find a power series centered at 0 for $g(x) = \frac{1+x}{(1-x)^2}$. (Hint: write $g(x)$ as a sum of two functions then use differentiation to find the series for each function). Determine the interval of convergence of the series you found.

Problem 4: Find power series by integration

1. Find a power series centered at 1 for $f(x) = \frac{1}{x}$. Determine the interval of convergence of the series you found.
2. Use the previous part to find a power series centered at 1 for $g(x) = \ln x$. (Hint: $g(x) = \int f(x)dx$.) Determine the interval of convergence of the series you found.

Problem 5: Using series for functions to find infinite sums

1. In the lecture we derived a series centered at 0 for $f(x) = \dfrac{x}{(1-x)^2}$. Use this result to find the sum of the series $\displaystyle\sum_{n=1}^{\infty} n\left(\frac{2}{3}\right)^n$.
2. In the lecture we derived a series centered at 0 for $f(x) = \ln(1+x)$. Use this result to find the sum of the series $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\frac{1}{2^n n}$.
3. In the lecture we derived a series centered at 0 for $f(x) = \arctan(x)$. Use this result to find the sum of the series $\displaystyle\sum_{n=0}^{\infty} (-1)^n\frac{1}{2n+1}$.