$$\frac{2s^2+10s}{(s^2-2s+5)(s+1)} = \frac{As+B}{s^2-2s+5} + \frac{C}{s+1}$$

$$2s^2+10s = (As+B)(s+1) + C(s^2-2s+5)$$

Plug in $s=-1$: $\quad -8 = 8C \longrightarrow \boxed{C=-1}$

Plug in $s=0$: $\quad 0 = B-5 \longrightarrow \boxed{B=5}$

Plug in $s=1$: $\quad 12 = (A+5)\cdot 2 - 4 \longrightarrow \boxed{A=3}$

$$\mathcal{L}^{-1}\left\{\frac{2s^2+10s}{(s^2-2s+5)(s+1)}\right\} = \mathcal{L}^{-1}\left\{\frac{3s+5}{s^2-2s+5}\right\} - \underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\}}_{e^{-x}}$$

$$\mathcal{L}^{-1}\left\{\frac{3s+5}{s^2-2s+1+4}\right\} = \mathcal{L}^{-1}\left\{\frac{3s+5}{(s-1)^2+4}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{3s-3+3+5}{(s-1)^2+4}\right\}$$

$$= 3\,\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+4}\right\} + \overset{4}{\cancel{8}}\cdot \mathcal{L}^{-1}\left\{\frac{\overset{2}{\cancel{1}}}{(s-1)^2+4}\right\}$$

$$= \boxed{3e^{x}\cos(2x) + 4\cdot e^{x}\sin(2x)}$$

Final answer: $\quad 3e^{x}\cos(2x) + 4e^{x}\sin(2x) - e^{-x}$

(3) $$\mathcal{L}^{-1}\left\{\frac{5s}{(s-2)^2}\right\} = \mathcal{L}^{-1}\left\{\frac{5s-10}{(s-2)^2}\right\} + \mathcal{L}^{-1}\left\{\frac{10}{(s-2)^2}\right\}$$

$$= 5\cdot\mathcal{L}^{-1}\left\{\frac{s-2}{(s-2)^2}\right\} + 10\,\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\}$$

$$= 5\cdot\mathcal{L}^{-1}\left\{\frac{1}{(s-2)}\right\} + 10\,\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\} \longrightarrow F(s-2)$$

$$= 5\,\mathcal{L}^{-1}\left\{\frac{1}{(s-2)}\right\} + 10\,\mathcal{L}^{-1}\left\{\frac{1}{(s-2)^2}\right\} \longrightarrow F(s-2)$$

where $F(s) = \frac{1}{s^2}$

$$= \boxed{5 \cdot e^{2x} + 10x\,e^{2x}}$$

E.g.3 ① $y'' - 4y' + 4y = x^3 e^{2x}$; $y(0) = y'(0) = 0$

$$\mathcal{L}\{y''\} - 4\,\mathcal{L}\{y'\} + 4\,\mathcal{L}\{y\} = \mathcal{L}\{x^3 e^{2x}\}$$

$$s^2 Y(s) - \cancel{s\,y(0)}^{0} - \cancel{y'(0)}^{0} - 4\cdot\left[s Y(s) - \cancel{y(0)}^{0}\right] + 4Y(s) = \frac{6}{(s-2)^4}$$

$$s^2 Y(s) - 4s Y(s) + 4Y(s) = \frac{6}{(s-2)^4}$$

$$Y(s)\left[s^2 - 4s + 4\right] = \frac{6}{(s-2)^4}$$

$$Y(s)\left[(s-2)^2\right] = \frac{6}{(s-2)^4}$$

$$\longrightarrow Y(s) = \frac{6}{(s-2)^6}\,.$$ Solution $y = \mathcal{L}^{-1}\{Y(s)\}$

$$y = \mathcal{L}^{-1}\left\{\frac{6}{(s-2)^6}\right\} = \frac{6}{5!}\cdot\mathcal{L}^{-1}\left\{\frac{\cancel{1}\,5!}{(s-2)^6}\right\}$$

$$\boxed{y = \frac{6}{5!}\,e^{2x}\cdot x^5}$$

② $y'' - 2y' + 5y = -8e^{-x}$; $y(0) = 2$; $y'(0) = 12$.

$$\mathcal{L}\{y''\} - 2\,\mathcal{L}\{y'\} + 5\,\mathcal{L}\{y\} = -8\,\mathcal{L}\{e^{-x}\}$$

$$s^2 Y(s) - s\,y(0) - y'(0) - 2\left[s Y(s) - y(0)\right] + 5Y(s) = -8\cdot\frac{1}{s+1}$$

$$s^2 Y(s) - 2s - 12 - 2sY(s) + 4 + 5Y(s) = \frac{-8}{s+1}$$

$$Y(s)\left[s^2 - 2s + 5\right] = \underbrace{-\frac{8}{s+1} + 2s + 8}_{\frac{-8 + (2s+8)(s+1)}{} = \frac{2s^2 + 10s}{}}$$