$$\frac{-8 + (2s+8)(s+1)}{s+1} = \frac{2s^2+10s}{s+1}$$

$$Y(s)\left[s^2-2s+5\right] = \frac{2s^2+10s}{s+1}$$

$$Y(s) = \frac{2s^2+10s}{(s+1)(s^2-2s+5)}$$

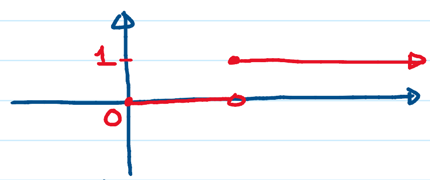
$$\rightarrow y = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{2s^2+10s}{(s+1)(s^2-2s+5)}\right\}$$

$$\rightarrow \boxed{y = 3e^x\cos(2x) + 4e^x\sin(2x)}$$

Translation in x Property.

Unit Step Function (Heaviside Function)

$$\boxed{\mathcal{U}(x-a)} = \begin{cases} 0 : & 0 \le x < a \\ 1 : & x \ge a \end{cases}$$



$\mathcal{U}(x-a)$ is "off" on $[0,a)$; it is "on" on $[a,\infty)$

$$\boxed{1-\mathcal{U}(x-a)} = \begin{cases} 1 : & 0 \le x < a \\ 0 : & x \ge a \end{cases}$$

$$\boxed{\mathcal{U}(x-a)-\mathcal{U}(x-b)} = \begin{cases} 0 : & 0 \le x < a \\ 1 : & a \le x < b \\ 0 : & x \ge b \end{cases}$$

E.g.

$$f(x) = \begin{cases} g(x) : & 0 \le x < a \\ h(x) : & a \le x < b \\ j(x) : & x \ge b \end{cases}$$

$$f(x) = \big(1 - \mathcal{U}(x-a)\big)\, g(x) + \big(\mathcal{U}(x-a) - \mathcal{U}(x-b)\big)\, h(x)$$
$$+ \ \mathcal{U}(x-b)\, j(x)$$

E.g.4 $f(x) = \left[1 - \mathcal{U}(x-2)\right] \cdot 3 + \left[\mathcal{U}(x-2) - \mathcal{U}(x-5)\right] \cdot 1$
$$+ \left[\mathcal{U}(x-5) - \mathcal{U}(x-7)\right] \cdot x + \mathcal{U}(x-7) \cdot x^2$$

Translation in x Property.

Laplace transform of a unit step function:

$$\boxed{\mathcal{L}\{\mathcal{U}(x-a)\} = \frac{e^{-as}}{s}}$$

Translation in x property – Form 1:

$$\boxed{\mathcal{L}\{ f(x-a) \cdot \mathcal{U}(x-a) \} = e^{-as} \cdot F(s)}$$

where $F(s) = \mathcal{L}\{f(x)\}$

E.g. $\mathcal{L}\{ \sin(x-2) \cdot \mathcal{U}(x-2) \} = e^{-2x} \cdot \dfrac{1}{s^2+1}$

In practice, we more likely have to find $\mathcal{L}\{ \sin x \, \mathcal{U}(x-2) \}$.

$\longrightarrow$ Translation in x property – Form 2:

$$\boxed{\mathcal{L}\{ g(x) \cdot \mathcal{U}(x-a) \} = e^{-as} \cdot G(s)}$$

where $G(s)$ is the laplace transform of $g(x+a)$

Eg. $\mathcal{L}\{ \sin x \cdot \mathcal{U}(x-2) \} \overset{?}{=} e^{-as} \cdot G(s)$

where $G(s)$ is the Laplace transform of $\sin(x+2)$

$\longrightarrow$ Find $\mathcal{L}\{ \sin(x+2) \}$