$\longrightarrow$ Find $\mathcal{L}\{\sin(x+2)\}$

$$= \mathcal{L}\{\sin x \cos 2 + \sin 2 \cos x\}$$

$$= \cos 2 \, \mathcal{L}\{\sin x\} + \sin 2 \, \mathcal{L}\{\cos x\}$$

$$= \frac{\cos 2}{s^2+1} + \frac{s \sin 2}{s^2+1}$$

$$\mathcal{L}\{\sin x \cdot \mathcal{U}(x-2)\} = e^{-2s} \cdot \left(\frac{\cos 2}{s^2+1} + \frac{s \sin 2}{s^2+1} \right)$$

<u>E.g.5</u> ①
$$\mathcal{L}\{(x-1)^2 \cdot \mathcal{U}(x-1)\} = e^{-s} \cdot \mathcal{L}\{x^2\}$$
$$= e^{-s} \cdot \frac{2}{s^3}$$

②
$$\mathcal{L}\{x^2 \cdot \mathcal{U}(x-1)\} = e^{-s} \cdot \mathcal{L}\{(x+1)^2\}$$
$$= e^{-s} \cdot \mathcal{L}\{x^2 + 2x + 1\}$$
$$= e^{-s} \cdot \left(\frac{2}{s^3} + \frac{2}{s^2} + \frac{1}{s} \right)$$

③
$$\mathcal{L}\{\sin x \cdot \mathcal{U}(x - \tfrac{\pi}{2})\} = e^{-\frac{\pi}{2}s} \cdot \mathcal{L}\{\sin(x + \tfrac{\pi}{2})\}$$
$$= e^{-\frac{\pi}{2}s} \cdot \mathcal{L}\{\sin x \cancelto{0}{\cos \tfrac{\pi}{2}} + \cancelto{1}{\sin \tfrac{\pi}{2}} \cos x\}$$
$$= e^{-\frac{\pi}{2}s} \cdot \mathcal{L}\{\cos x\}$$
$$= e^{-\frac{\pi}{2}s} \cdot \frac{s}{s^2+1}$$

Tuesday, August 6, 2019 9:49 AM

E.g. 6.

$$f(x) = \left(1 - \mathcal{U}(x-2)\right)\cdot 3 + \left(\mathcal{U}(x-2) - \mathcal{U}(x-5)\right)\cdot 1$$
$$+ \left(\mathcal{U}(x-5) - \mathcal{U}(x-7)\right)x + \mathcal{U}(x-7)\cdot x^2$$

$$\mathcal{L}\{f(x)\} = 3\cdot\mathcal{L}\{1\} - 3\,\mathcal{L}\{\mathcal{U}(x-2)\} + \mathcal{L}\{\mathcal{U}(x-2)\} - \mathcal{L}\{\mathcal{U}(x-5)\}$$
$$+ \mathcal{L}\{x.\,\mathcal{U}(x-5)\} - \mathcal{L}\{x.\,\mathcal{U}(x-7)\} + \mathcal{L}\{x^2.\,\mathcal{U}(x-7)\}$$
$$= \frac{3}{s} - 3.\frac{e^{-2s}}{s} + \frac{e^{-2s}}{s} - \frac{e^{-5s}}{s} + e^{-5s}.\mathcal{L}\{x+5\} - e^{-7s}.\mathcal{L}\{x+7\}$$
$$+ e^{-7s}.\mathcal{L}\{(x+7)^2\}$$
$$= \frac{3}{s} - 2.\frac{e^{-2s}}{s} - \frac{e^{-5s}}{s} + e^{-5s}.\left(\frac{1}{s^2} + \frac{5}{s}\right) - e^{-7s}.\left(\frac{1}{s^2} + \frac{7}{s}\right)$$
$$+ e^{-7s}.\left(\frac{2}{s^3} + \frac{14}{s^2} + \frac{49}{s}\right) = \ldots \text{simplify.}$$

Translation in x property: $\mathcal{L}\{f(x-a)\,\mathcal{U}(x-a)\} = e^{-as}.\,F(s)$

$$\longrightarrow \boxed{\mathcal{L}^{-1}\{e^{-as}.\,F(s)\} = f(x-a)\,\mathcal{U}(x-a)}$$

E.g. 7.

$a = \frac{\pi}{2}$ $\quad F(s)$

① $$\mathcal{L}^{-1}\left\{\frac{s\cdot e^{-\frac{\pi s}{2}}}{s^2+4}\right\} = \mathcal{L}^{-1}\left\{e^{-\frac{\pi}{2}s}.\frac{s}{s^2+4}\right\} = \boxed{\cos\left(2\left(x-\tfrac{\pi}{2}\right)\right)\cdot\mathcal{U}\left(x-\tfrac{\pi}{2}\right)}$$

② $$\mathcal{L}^{-1}\left\{\frac{(1+e^{-2s})^2}{s+2}\right\} = \mathcal{L}^{-1}\left\{\frac{1+2e^{-2s}+e^{-4s}}{s+2}\right\}$$
$$= \mathcal{L}^{-1}\left\{\frac{1}{s+2}\right\} + 2\,\mathcal{L}^{-1}\left\{\frac{e^{-2s}}{s+2}\right\} + \mathcal{L}^{-1}\left\{\frac{e^{-4s}}{s+2}\right\}$$
$$= e^{-2x} + 2\cdot e^{-2(x-2)}\cdot\mathcal{U}(x-2) + e^{-2(x-4)}\cdot\mathcal{U}(x-4)$$