Tuesday, August 6, 2019 11:02 AM

$$\mathcal{L}\{y'\} = sY(s) - y(0) = \underbrace{sY(s)}$$

1^{st} deri $\rightarrow Y(s) + sY'(s)$

E.g.2 :

$$y'' + 2xy' - 4y = 1; \quad y(0) = y'(0) = 0$$

(Here: $Y(s) = \mathcal{L}\{y\}$)

$$\mathcal{L}\{y''\} + 2\,\mathcal{L}\{xy'\} - 4\,\mathcal{L}\{y\} = \mathcal{L}\{1\}$$

$$s^2Y(s) - s\cancel{y(0)}^{\,0} - \cancel{y'(0)}^{\,0} \qquad\qquad - 4Y(s) = \frac{1}{s}$$

$$s^2Y(s) - 2\big(Y(s) + sY'(s)\big) \qquad\qquad - 4Y(s) = \frac{1}{s}$$

$$s^2Y(s) - 2Y(s) - 2sY'(s) - 4Y(s) = \frac{1}{s}.$$

$$-2sY'(s) + (s^2-6)Y(s) = \frac{1}{s}.$$

$$Y'(s) - \frac{s^2-6}{2s}Y(s) = -\frac{1}{2s^2}.$$

$$I = e^{\int -\frac{s^2-6}{2s}ds} = e^{-\int\frac{s^2-6}{2s}ds} = e^{-\int\left(\frac{s}{2}-\frac{3}{s}\right)ds}$$

$$I = e^{-\frac{s^2}{4}+3\ln(s)} = e^{-\frac{s^2}{4}}\cdot e^{3\ln(s)} = s^3\cdot e^{-\frac{s^2}{4}}$$

Multiply both sides by I:

$$\frac{d}{ds}\left[I\cdot Y(s)\right] = -\frac{1}{2s^2}\cdot s^3\cdot e^{-\frac{s^2}{4}}$$

$$\frac{d}{ds}\left[I\cdot Y(s)\right] = -\frac{s}{2}\cdot e^{-\frac{s^2}{4}}$$

$$u = -\frac{s^2}{4} \rightarrow du = -\frac{s}{2}ds$$

$$I\cdot Y(s) = -\frac{1}{2}\int s\cdot e^{-\frac{s^2}{4}}ds$$

$$I \cdot Y(s) = \int e^{u}\,du = e^{u} + C$$

$$I \cdot Y(s) = e^{-\frac{s^2}{4}} + C$$

$$s^3 \cdot e^{-\frac{s^2}{4}} \cdot Y(s) = e^{-\frac{s^2}{4}} + C$$

$$Y(s) = \frac{1}{s^3} + \frac{C}{s^3 e^{-s^2/4}} = \frac{1}{s^3} + \frac{C}{s^3} \cdot e^{s^2/4}$$

Since $\lim_{s \to \infty} Y(s) = 0$, $C = 0$

$$Y(s) = \frac{1}{s^3}.$$

$$\longrightarrow \quad y = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \boxed{\frac{x^2}{2}}.$$