Lecture 16

Tuesday, August 6, 2019 10:39 AM

Derivatives of Laplace Transform.

$$\mathcal{L}\{x^n \cdot f(x)\} = (-1)^n \cdot \frac{d^n F}{ds^n}(s)$$

E.g. 1: ① $\mathcal{L}\{x \cdot \sin(bx)\}$

Step 1: $F(s) = \mathcal{L}\{\sin(bx)\} = \frac{b}{s^2+b^2}$

1st derivative $= \frac{-b}{(s^2+b^2)^2} \cdot 2s$

$b(s^2+b^2)^{-1}$

Step 2: $\mathcal{L}\{x \cdot \sin(bx)\} = -1 \cdot \frac{(-2sb)}{(s^2+b^2)^2} = \frac{2sb}{(s^2+b^2)^2}$

② $\mathcal{L}\{x^2 \cdot \cos(x)\}$

Step 1: $F(s) = \mathcal{L}\{\cos x\} = \frac{s}{s^2+1}$

1st $\frac{(s^2+1)\cdot 1 - s \cdot 2s}{(s^2+1)^2}$

$\frac{-s^2+1}{(s^2+1)^2}$

2nd $\frac{-2s(s^2+1)^2 - (-s^2+1)\cdot 2(s^2+1)\cdot 2s}{(s^2+1)^{4}}$

$\frac{-2s(s^2+1) - 4s(-s^2+1)}{(s^2+1)^3}$

$\frac{-2s^3-2s+4s^3-4s}{(s^2+1)^3}$

Step 2: $\mathcal{L}\{x^2 \cdot \cos(x)\} = (-1)^2 \cdot \left(\frac{2s^3-6s}{(s^2+1)^3}\right)$

$= \frac{2s^3-6s}{(s^2+1)^3}$

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③ $\mathcal{L}\{x \cdot e^{-3x} \cdot \cos(3x)\}$

Step 1: Find $\mathcal{L}\{e^{-3x} \cdot \cos(3x)\}$ → Translation in s property.

$\mathcal{L}\{\cos(3x)\} = \frac{s}{s^2+9}$

→ $\mathcal{L}\{e^{-3x}\cos(3x)\} = \frac{s+3}{(s+3)^2+9}$

1st $\frac{1\cdot((s+3)^2+9) - (s+3)(2(s+3))}{[(s+3)^2+9]^2}$

$\frac{(s+3)^2+9-2(s+3)^2}{[(s+3)^2+9]^2}$

Step 2: $\mathcal{L}\{x \cdot e^{-3x} \cdot \cos(3x)\} = (-1)^1 \cdot \frac{9-(s+3)^2}{}$

$[(s+3)^2+9]^2$

Step 2: $\mathcal{L}\{x \cdot e^{-3x} \cdot \cos(3x)\} = (-1)^1 \cdot \dfrac{9-(s+3)^2}{[(s+3)^2+9]^2}$

$$= \frac{(s+3)^2-9}{[(s+3)^2+9]^2}.$$