Due at the beginning of class on the day of Test 1

Direction: Solve the problems in this assignment on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, each part of the problem and include the problems in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the assignment. Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Definitions and terminology

Determine the order of the ODE and determine whether it is linear or nonlinear. If it is nonlinear, explain why.

1. $xy''' - (y')^4 + y = 0$
2. $x^5 \dfrac{d^4y}{dx^4} - x^3 \dfrac{d^2y}{dx^2} + 6y = 0$
3. $\sqrt{1-y}\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} = 0$
4. $y'' + y' + y = \cos(x+y)$.

Problem 2: Verify solution

Verify that the indicated function is a solution of the given ODE.

1. $2y' + y = 0, \quad y = e^{-x/2}$
2. $y'' - 6y' + 13y = 0, \quad y = e^{3x}\cos(2x)$
3. $(y-x)y' = y - x + 8, \quad y = x + 4\sqrt{x+2}$.

Problem 3: Implicit solution

Determine whether the relation $G(x,y) = x^2y - y - 7 = 0$ is an implicit solution of the ODE

$$(x^2-1)\frac{dy}{dx} + 2xy = 0 \text{ on the interval } I = (1,\infty).$$

Problem 4: Family of solutions

1. Verify that the 2-parameter family of functions $y = c_1 \sin x + c_2 \cos x$ is a family of solutions to the ODE

$$\frac{d^2y}{dx^2} + y = 0.$$

2. Verify that the 2-parameter family of functions $y = c_1e^{2x} + c_2xe^{2x}$ is a family of solutions to the ODE

$$y'' - 4y' + 4y = 0.$$

Problem 5: Find a particular solution

1. Given that $y = c_1e^x + c_2e^{-x}$ is a 2-parameter family of solutios to the ODE $y'' - y = 0$. Find the particular solution that satisfies the initial condition $y(0) = 1$ and $y'(0) = 2$.

Problem 6: Find a differential equations

Find a differential equation whose solution is the given n-parameter family:

1. $y = c_1\cos(3x) + c_2\sin(3x)$
2. $y = c_1\sin x + c_2\cos x + x^2$.

Problem 7: Find a solution

Find the value(s) of m for which the function $y = e^{mx}$ is a solution of the ODE

$$y''' + 3y'' + 2y = 0.$$

Problem 8: How do differential equations arise?

Write a differential equation that fits the physical description:

1. The rate of change of a population of human societies, or animal species or bacteria colonies, etc. at time t is proportional to the population itself at time t. Let $y(t)$ denote the population at time t and k be the constant of proportionality.
2. The rate of change of the mass $y(t)$ of salt at time t is proportional to the square of the mass of salt present at time t. Let k be the constant of proportionality.
3. The net force F acting on a falling object of mass m equals to the difference between gravity and the resistive force of air. The resistive force of air is proportional to the velocity v of the object. Let $y(t)$ be the position of the object at time t. Constant of proportionality is k. Acceleration due to gravity is g. (Hint: $F = mg - kv$. By Newton's second law, $F = ma$. Thus, $ma = mg - kv$. Then write the acceleration $a(t)$ and the velocity $v(t)$ as derivatives of $y(t)$.)