Homework 10: Variation of Parameters

Due at the beginning of class on the day of Test 2

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Solve a nonhomogeneous equation

Use variation of parameters to find a particular solution of the equation. Then find the general solution of the equation.

1. $y'' + y = \sec x$
2. $y'' + 4y = \csc^2(2x)$.

Problem 2: Solve a nonhomogeneous equation

Use variation of parameters to find a particular solution of the equation. Then find the general solution of the equation.

1. $y'' + 3y' + 2y = \dfrac{1}{1+e^x}$
2. $y'' + 2y' + y = e^{-x}\ln x$.

Problem 3: Solve a nonhomogeneous equation - Variable Coefficients

Given that $y_1 = x^{-1/2}\cos x$ and $y_2 = x^{-1/2}\sin x$ are linearly independent solutions of the associated homogeneous equation. Use variation of parameters to find a particular solution and find the general solution of the equation.

$$x^2y'' + xy' + (x^2 - \frac{1}{4})y = x^{3/2}.$$

Problem 4: Solve a nonhomogeneous equation - Variable Coefficients

Given that $y_1 = \cos(\ln x)$ and $y_2 = \sin(\ln x)$ are linearly independent solutions of the associated homogeneous equation. Use variation of parameters to find a particular solution and find the general solution of the equation.

$$x^2y'' + xy' + y = \sec(\ln x).$$

Problem 5: Cauchy-Euler Equation

A linear second order equation (with variable coefficients) of the form

$$ax^2y'' + bxy' + cy = g(x)$$

where a, b, c are constants, $a \neq 0$ is called a **Cauchy-Euler** equation. The characteristic equation for the associated homogeneous equation of a Cauchy-Euler equation is given by

$$am^2 + (b-a)m + c = 0.$$

If this equation has distinct real roots m_1 and m_2, then the complementary function is $y_c = c_1x^{m_1} + c_2x^{m_2}$.
Given the Cauchy-Euler equation:

$$x^2y'' + xy' - y = \ln x.$$

1. Find and solve the characteristic equation. Use the roots to find the complementary function y_c. This gives rise to two linearly independent solutions $y_1 = x^{m_1}$ and $y_2 = x^{m_2}$ of the associated homogeneous equation.
2. Use y_1, y_2 from part 1 and the method of variation of parameter to find a particular solution to the equation. Write down the general solution to the equation.