Due at the beginning of class on the day of Test 2

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment. Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Shifting the index of summation and combine power series

Shift the index of summation to rewrite the given expression as a single power series whose generic term involves x^n

1. $\displaystyle\sum_{n=1}^{\infty} 2na_n x^{n-1} + \sum_{n=0}^{\infty} 6a_n x^{n+1}$

2. $\displaystyle\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - 2\sum_{n=1}^{\infty} na_n x^n + \sum_{n=0}^{\infty} a_n x^n$

Problem 2: Power Series Solution - First Order Equation

1. Find the first four nonzero terms of a power series solution centered at 0 for the equation $y' + (x+2)y = 0$.

2. Solve the differential equation $y' + 2xy = 0$ assuming a power series solution centered at 0. Your answer should include a general formula for the solution.

Problem 3: Power Series Solution - Second Order Equation

Given the equation $y'' - y = 0$.

1. Find the general solution using the method of characteristic equation.

2. Find the general solution assuming a power series solution centered at 0. Compare the series solution with the solution you obtain from part 1.

Problem 4: Power Series Solution - Second Order Equation

1. Find the first four nonzero terms of a power series solution centered at 0 for the equation $y'' - x^2 y = 0$.

2. Find the first four nonzero terms of a power series solution centered at 0 for the equation $y'' - x^2 y' + y = 0$.

Problem 5: Power Series Solution - Second Order IVP

Find the first four nonzero terms of a power series solution centered at 0 for the IVP:

$$(x+1)y'' - y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

Answers:

Problem 2: 1. $y = a_0\left(1 - 2x + \dfrac{3}{2}x^2 - \dfrac{1}{3}x^3 + \ldots\right)$. 2. $y = a_0\displaystyle\sum_{n=0}^{\infty}\dfrac{(-1)^n}{n!}x^{2n}$.

Problem 4: 1. $y = a_0\left(1 + \dfrac{1}{12}x^4 + \ldots\right) + a_1\left(x + \dfrac{1}{20}x^5 + \ldots\right)$. 2. $a_0\left(1 - \dfrac{1}{2}x^2 + \ldots\right) + a_1\left(x - \dfrac{1}{6}x^3 + \ldots\right)$.

Problem 5: $y = x + \dfrac{1}{6}x^3 - \dfrac{1}{12}x^4 + \dfrac{7}{120}x^5$.