Due at the beginning of class on the day of Test 2

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Find the Laplace transform of a function by definition

Use the definition of the Laplace transform $\mathscr{L}\{f(x)\} = \int_0^\infty e^{-sx} f(x)dx$ to find the Laplace transform of the given function $f(x)$

1. $f(x) = xe^{4x}$
2. $f(x) = x\cos(x)$.

Problem 2: Laplace transform of a piecewise function by definition

Use the definition of the Laplace transform $\mathscr{L}\{f(x)\} = \int_0^\infty e^{-sx} f(x)dx$ to find the Laplace transform of the given piecewise function $f(x)$

1. $f(x) = \begin{cases} -1 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$
2. $f(x) = \begin{cases} \sin x & 0 \le x < \pi \\ 0 & x \ge \pi \end{cases}$

Problem 3: Find Laplace Transforms of functions using Table and Linearity

Use the table of basic Laplace transforms and linearity property to find the Laplace transform of the function:

1. $f(x) = 2x^4$
2. $f(x) = (x+1)^3$
3. $f(x) = 1 + e^{4x}$
4. $f(x) = (1 + e^{2x})^2$
5. $f(x) = 4x^2 - 5\sin(3x)$
6. $f(x) = x^4 - x^2 - x + \cos(\sqrt{2}x)$

Problem 4: Find Laplace Transforms of functions using Table and Linearity

First use a trig identity, and then use the table of basic Laplace transforms and linearity property to find the Laplace transform of the function:

1. $f(x) = e^x \sinh(x)$
2. $f(x) = \sin(2x)\cos(2x)$
3. $f(x) = 10\cos\left(x - \frac{\pi}{6}\right)$

Hint: For part 1 use the definition of $\sinh(x)$. For part 2 use the double-angle identity. For part 3 use the cosine of a difference formula.

Problem 5: First Translation Theorem

1. Read and explain the proof of Theorem 7.3.1 (Pg 271) in the book: if $\mathscr{L}\{f(x)\} = F(s)$, then
$$\mathscr{L}\{e^{ax} f(x)\} = F(s-a).$$
2. Apply the theorem and the table of basic Laplace transforms to come up with the formula for:

(a) $\mathscr{L}\{e^{ax}\sin(bx)\}$ (b) $\mathscr{L}\{e^{ax}\cos(bx)\}$ (c) $\mathscr{L}\{e^{ax}t^n\}$ (d) $\mathscr{L}\{e^{ax}\sinh(bx)\}$ (e) $\mathscr{L}\{e^{ax}\cosh(bx)\}$