Due at the beginning of class on the day of Test 3

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Find Laplace transforms using the translation in s property

Find the Laplace transform

1. $\mathscr{L}\left\{x^3e^{-2x}\right\}$
2. $\mathscr{L}\left\{x\left(e^x+e^{2x}\right)^2\right\}$.
3. $\mathscr{L}\left\{e^x\sin(3x)\right\}$
4. $\mathscr{L}\left\{\left(1-e^x+3e^{-4x}\right)\cos(5x)\right\}$.

Problem 2: Find inverse Laplace transforms using the translation in s property

Find the inverse Laplace transforms:

1. $\mathscr{L}^{-1}\left\{\dfrac{1}{(s+2)^3}\right\}$
2. $\mathscr{L}^{-1}\left\{\dfrac{2s+5}{s^2+6s+34}\right\}$
3. $\mathscr{L}^{-1}\left\{\dfrac{s}{(s+1)^2}\right\}$.

Problem 3: Solve an IVP by using the Laplace transform

1. $y''-6y'+9y=x$, $y(0)=0; y'(0)=1$
2. $y''-y'=e^x\cos(x)$, $y(0)=0$, $y'(0)=0$.

Problem 4: Find Laplace transform using translation in x property

Find the Laplace transform:

1. $\mathscr{L}\left\{(x-1)\mathcal{U}(x-1)\right\}$
2. $\mathscr{L}\left\{x\mathcal{U}(x-2)\right\}$
3. $\mathscr{L}\left\{\cos(2x)\mathcal{U}(x-\pi)\right\}$
4. $\mathscr{L}\left\{e^{2-x}\mathcal{U}(x-2)\right\}$

Problem 5: Find Laplace transform of a piecewise function using translation in x property

Express the given piecewise function using unit step functions and find its Laplace transform:

1. $f(x)=\begin{cases}0 & 0\le x<1\\ x^2 & x\ge 1.\end{cases}$
2. $f(x)=\begin{cases}0 & 0\le x<1\\ 2 & 1\le x<2\\ 1 & 2\le x<3\\ 3 & x\ge 3.\end{cases}$
3. $f(x)=\begin{cases}\sin x & 0\le x<2\pi\\ 0 & x\ge 2\pi.\end{cases}$

Problem 6: Find inverse Laplace transform using translation in x property

Find the inverse Laplace transform:

1. $\mathscr{L}^{-1}\left\{\dfrac{e^{-\pi s}}{s^2+1}\right\}$
2. $\mathscr{L}^{-1}\left\{\dfrac{e^{-s}}{s(s+1)}\right\}$

Problem 7: Solve an IVP using Laplace transform

1. Solve the IVP:

$$y'' + y = f(x), y(0) = 0, y'(0) = 1,$$

where $g(x) = \begin{cases} 0 & 0 \le x < \pi \\ 1 & \pi \le x < 2\pi \\ 0 & x \ge 2\pi \end{cases}$.

2. Solve the IVP:

$$y'' + 3y' + 2y = g(x), y(0) = 2, y'(0) = -1,$$

where $g(x) = \begin{cases} e^{-x} & 0 \le x < 3 \\ 1 & x \ge 3 \end{cases}$.