HW2 - Separable Equations

Due at the beginning of class on the day of Test 1

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Solve a separable equation

Solve the given separable equation. Is the equation linear or nonlinear? Is your solution an implicit or an explicit solution? If it is implicit, can you solve for y explicitly?

1. $x\dfrac{dy}{dx} = \dfrac{1}{y^3}$
2. $dx + e^{3x}dy = 0$
3. $\dfrac{dy}{dx} = e^{3x+2y}$
4. $\dfrac{dy}{dx} = y - y^2$ (Hint: partial fractions decomposition)

Problem 2: Solve an initial value problem

Solve the initial value problem. Is the equation linear or nonlinear? Is your solution an implicit or an explicit solution? If it is implicit, can you solve for y explicitly?

1. $\dfrac{dy}{dx} = 2x\cos^2(y)$, $y(0) = \pi/4$
2. $y' = e^{2y}\cos(2x)$, $y(\pi/2) = 0$

Problem 3: Transform an equation to a separable equation

Solve the given equation

$$\frac{dy}{dx} = \frac{10}{(x+y)e^{x+y}} - 1.$$

Hint: First make the substitution $u = x + y$. Then $y = u - x$. Thus, $\frac{dy}{dx} = \frac{du}{dx} - 1$. Substitute this to the left side of the equation and substitute $u = x + y$ to the right side. Then solve.

Problem 4: Newton's Law of Cooling

This problem uses Newton's Law of Cooling, in particular, the solution function T of the differential equation in Example 4 of Lecture 3.

Blood plasma is stored at 40°F, i.e., $T(0) = 40$°F. It takes 45 minutes for the blood plasma to warm to 90°F, the required temperature for usage, if we place it in an oven at 120°F. How long will it take for the plasma to warm to 90°F if we set the oven temperature at 140°F?

Problem 5: Compound Interest

Let $P(t)$ be the amount at time t (in years) in a bank account with annual interest rate of r% compounded continuously. Then the function P satisfies the equation

$$\frac{dP}{dt} = \frac{r}{100}P.$$

1. Solve the equation for P. (Note that r is a constant.)
2. If the annual interest rate is $r = 5$% and the initial amount in the account is \$10000, find the amount in the account after 5 years? How long does it take for the amount of money in the account to double? Triple?
3. If a constant amount a is added to the account every year, how will the equation change? Solve the new equation for P.
4. Now suppose the constant amount added to the account every year is $a = \$1000$, find the amount in the account after 5 years.