Homework 3 - First Order Linear Equations

Due at the beginning of class on the day of Test 1

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Solve a linear first order equation

Solve the linear first order equations:

1. $\dfrac{dy}{dx} + 3x^2y = x^2$
2. $xy' - y = x^2 \sin x$
3. $\dfrac{dy}{dx} + \sec(x)y = \cos(x)$.

Problem 2: Solve an initial value problem

Silve the IVP:

1. $xy' + y = e^x$, $y(1) = 2$
2. $(x+1)\frac{dy}{dx} + y = \ln x$, $y(1) = 10$
3. $y' - (\sin x)y = 2\sin x$, $y(\pi/2) = 1$

Problem 3: Discontinuous forcing term

Apply the process in Example 3 of Lecture 4 to solve the linear equation with discontinuous forcing term $Q(x)$:

$$\frac{dy}{dx} + 2xy = Q(x), y(0) = 2 \text{ where}$$

$$Q(x) = \begin{cases} x, & \text{if } 0 \le x < 1 \\ 0, & \text{if } x \ge 1 \end{cases}$$

Problem 4: Discontinuous Coefficient

Consider the equation

$$\frac{dy}{dx} + P(x)y = 4x, y(0) = 3 \text{ where the coefficient } P(x) \text{ is the discontinuous function}$$

$$P(x) = \begin{cases} 2, & \text{if } 0 \le x \le 1 \\ -\frac{2}{x}, & \text{if } x > 1 \end{cases}$$

1. Find the general solution for $0 \le x \le 1$ and choose the constant in the solution so that the initial condition is satisfied.
2. Find the general solution for $x > 1$ and choose the constant in the solution so that the solution from the previous part and the solution in this part agree at $x = 1$.

Problem 5: An application - equation with discontinuous forcing term

The temperature at time t, $y(t)$ in units of 100°F of a room in the winter satisfies the differential equation

$$\frac{dy}{dt} = \begin{cases} -y+1, & \text{if heating unit is on} \\ -y, & \text{if heating unit is off} \end{cases}$$

Suppose the initial temperature is $y(0) = 0$ at 9a.m. (Let the time $t = 0$ corresponds to 9a.m., $t = 1$ corresponds to 10a.m. and so on.) The heating unit is on from 9 to 10a.m., off from 10 to 11a.m., on again from 11a.m. to noon and so on. What is the room temperature at noon? (Hint: the answer is 71.8°F.)

Problem 6: Free falling object with air resistance

The net force F acting on a free falling object of mass m equals to the difference between gravity and the resistive force of air. The resistive force of air is proportional to the velocity v of the object. Let $y(t)$ be the position of the object at time t. Constant of proportionality is k. Acceleration due to gravity is g. Then the equation for the net force is $F = mg - kv$. By Newton's second law, $F = ma$. Thus, we have the equation $ma = mg - kv$. Since $a = \frac{dv}{dt}$, we obtain a linear first order differential equation that the velocity v of the object must satisfy

$$m\frac{dv}{dt} = mg - kv.$$

Divide both sides by m and rearrange, we obtain

$$\frac{dv}{dt} + \frac{k}{m}v = g.$$

Solve this equation for the velocity $v(t)$ of the object, assume $v(0) = 0$. (Note that m, g and k are constants.) The **terminal velocity** v_∞ of the object is given by $v_\infty = \lim_{t\to\infty} v(t)$. Find v_∞ in terms of m, g and k.

Problem 7: Series Circuit

Let $i(t)$ denote the current at time t in a series circuit containing a resistor and an inductor. Kirchhoff's voltage law says that the sum of the voltage drop across the inductor ($L\frac{di}{dt}$) and the voltage drop across the resistor (iR) is the same as the impressed voltage E. This gives rise to a first order linear differential equation for the current $i(t)$

$$L\frac{di}{dt} + Ri = E.$$

Assume that $i(0) = 0$. Solve this equation for $i(t)$. (Note that L, R and E are constants.) The **steady-state current** is given by $i_\infty = \lim_{t\to\infty} i(t)$. Find i_∞.