Homework 4: Exact Equations

Due at the beginning of class on the day of Test 1

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Solving an exact equation

Verify that the equation is exact and find a 1-parameter family of solutions of the equation

1. $(2x-1)dx+(3y+7)dy$
2. $(2xy^2-3)dx+(2x^2y+4)dy=0$
3. $(x-y^3+y^2\sin x)dx=(3xy^2+2y\cos x)dy$
4. $\left(ye^{xy}-\dfrac{1}{y}\right)dx+\left(xe^{xy}+\dfrac{x}{y^2}\right)dy=0.$

Problem 2: Solving an IVP

Verify that the equation is exact and solve the initial value problem

1. $(y^2\cos x-3x^2y-2x)dx+(2y\sin x-x^3+\ln y)dy=0$, $y(0)=e$
2. $(e^xy+xe^xy)dx+(xe^x+2)dy=0$, $y(0)=-1$

Problem 3: Nonexact equations made exact

Show that the differential equation is not exact. Find an integrating factor to make it exact. Solve the equation.

1. $(2y^2+3x)dx+2xydy=0$
2. $xdx+(x^2y+4y)dy=0$, $y(4)=0$.

Problem 4: Orthogonal trajectories

Consider the family of hyperpolas $F(x,y)=xy=C$ where C is a parameter. The **orthogonal trajectories** of this family is the family of curves each of whose member is orthogonal (perpendicular) to every curve in the former family. Now, the slope at (x,y) for each curve in the family $F(x,y)=xy=C$ is given by

$$\frac{dy}{dx}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}.$$

Hence, the the slope for each curve in the orthogonal trajectories must be

$$\frac{dy}{dx}=\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial x}}(\text{ negative reciprocal}).$$

Rearrange the above equation, it follows that the curves in the orthogonal trajectories must satisfy the equation

$$\frac{\partial F}{\partial y}dx-\frac{\partial F}{\partial x}dy=0.$$

Find the partial derivatives $\partial F/\partial x$ and $\partial F/\partial y$ of the function $F(x,y)=xy$ and plug them into the above equation. Then solve the equation to find a 1-parameter family of solutions.
Your solution will be the orthogonal trajectories of the family of hyperbolas $F(x,y)=xy=C$.