Homework 5: Solution by Substitutions

Due at the beginning of class on the day of Test 1

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Solve a homogeneous differential equation

Explain why the equation is homogeneous and use the substitution method to solve.

1. $(y^2 + yx)dx - x^2 dy = 0$
2. $\dfrac{dy}{dx} = \dfrac{y-x}{y+x}$
3. $\dfrac{dy}{dx} = \dfrac{y^2 + x\sqrt{x^2+y^2}}{xy}$
4. $\dfrac{dy}{dx} = \dfrac{x\sec(y/x) + y}{x}$.

Problem 2: Solve an IVP

Solve the given IVP

1. $(x + ye^{y/x})dx - xe^{y/x}dy = 0$, $y(1) = 0$
2. $\dfrac{dy}{dx} = \dfrac{y(\ln y - \ln x + 1)}{x}$, $y(1) = e$.

Problem 3: Solve a Bernoulli's equation

Solve the Bernoulli's equation:

1. $x\dfrac{dy}{dx} + y = \dfrac{1}{y^2}$
2. $\dfrac{dy}{dx} = y(xy^3 - 1)$.

Problem 4: Equation of the form $\dfrac{dy}{dx} = G(ax+by)$

1. Solve the equation $\dfrac{dy}{dx} = \sqrt{x+y} - 1$. Hint: Let $u = x + y$. Then $\dfrac{du}{dx} = 1 + \dfrac{dy}{dx}$. So, $\dfrac{dy}{dx} = \dfrac{du}{dx} - 1$. Substitute this to the original equation to get a separable equation for the function u. Solve for u and then solve for y.
2. Solve the equation $\dfrac{dy}{dx} = (x - y + 5)^2$. Hint: Let $u = x - y$ and follow the strategy of the previous problem.

Problem 5: Population growth

Consider the logistic equation that is used to model the growth of certain population:

$$\frac{dP}{dt} = P(a - bP)$$

where a and b are positive constants. This is a Bernoulli's equation.

1. Use the method in the lecture to solve the equation for the function P.
2. Suppose that $P(0) = P_0$. If $P_0 > 0$, show that $\lim\limits_{t\to\infty} P(t) = \dfrac{a}{b}$.