Due at the beginning of class on the day of Test 1

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: nth-order initial value problem

Determine the largest interval $I = (a, b)$ for which the Existence and Uniqueness Theorem guarantees the existence of a unique solution on (a, b) to the initial value problem.

1. $xy''' - 3y' + e^x y = x^2 - 1$; $y(-2) = 1$, $y'(-2) = 0$, $y''(-2) = 2$
2. $x\sqrt{x+1}y''' - y' + xy = 0$; $y(1/2) = y'(1/2) = -1$, $y''(1/2) = 1$
3. $(x^2 - 1)y''' + e^x y = \ln x$; $y(3/4) = 1$, $y'(3/4) = y''(3/4) = 0$.

Problem 2: Find the unique solution

Consider the initial value problem

$$\text{Solve: } x^2y'' - xy' + y = 0$$
$$\text{Subject to: } y(1) = 3, y'(1) = -1.$$

1. Determine the largest interval $I = (a, b)$ for which the IVP has a unique solution.
2. Given that the 2-parameter family of functions $y = c_1x + c_2x \ln x$ is the general solution of the equation $x^2y'' - xy' + y = 0$. Find a member of the family that is the unique solution of the IVP.

Problem 3: Linearly dependent/independent

Determine whether the given collection of functions is linearly dependent or linearly independent on the interval. Justify the answer.

1. $\{x, x^2, 4x - 3x^2\}$ on $I = (-\infty, \infty)$
2. $\{\sin^2(x), \cos^2(x), 7\}$ on $I = (-\infty, \infty)$
3. $\{x, xe^x, 1\}$ on $I = (-\infty, \infty)$

Problem 4: Fundamental set of solutions for a homogeneous equation

Verify that the given collection of functions form a fundamental set of solutions for the equation on the indicated interval. Form the general solution to the equation.

1. $y'' - y' - 12y = 0; \{e^{-3x}, e^{4x}\}, (-\infty, \infty)$
2. $x^2y'' - 6xy' + 12y = 0; \{x^3, x^4\}, (0, \infty)$.

Problem 5: General solution to a nonhomogeneous equation

Verify that the given function is a particular solution of the nonhomogeneous equation. The fundamental set of solutions for the associated homogeneous equation is given in the previous exercise. Find the general solution to the nonhomogeneous equation.

1. $y'' - y' - 12y = 24e^x; y_p(x) = -2e^x$.
2. $x^2y'' - 6xy' + 12y = x^2 + 6x + 4; y_p(x) = \dfrac{x^2}{2} + x + \dfrac{1}{3}$.

Problem 6: Working with differential operators

1. Let $L = D^2 + 2D - 8$ and $y = e^{-2x}$. Find Ly.
2. Let $L = D^3 + D + x$ and $y = \sin x$. Find Ly.
3. Let $L = D^2 - \dfrac{1}{x}D + \dfrac{1}{x^2}$ and $y = x \ln x$. Find Ly.

Problem 7: Factor a differential operator

1. Show that the operator D^2+6D+9 is the same as the operator $(D+3)^2$, that is, show that for any differentiable function y, we have
$$(D^2 + 6D + 9)y = (D + 3)^2 y.$$
2. Let $y = xe^{-3x}$. Find $(D + 3)^2 y$.

Problem 8: Express equations using differential operators

Find the nth-order linear differential operator L such that the given differential equation can be expressed in the form $L(y) = g(x)$. Factor L.

1. $y'' - 9y = x^2 - 2x$
2. $y''' + 4y'' + 3D = x^2 \cos x - 3x$.