Homework 7: Reduction of Order

Due at the beginning of class on the day of Test 1

Direction: Solve the problems in this worksheet on separate sheets of paper. Write your solution neatly. Use standard size paper. Clearly label each problem, and include each problem in the correct order. No ragged edges. Staple multiple pages. At the top of the first page put your name, Math 2320, and the title of the homework assignment.Show all work to justify your answer. Answer with insufficient work will receive no credit.

Problem 1: Find a second solution by reduction of order - constant coefficients

The given function $y_1(x)$ is a solution of the given differential equation. Use the reduction of order method to find a second solution $y_2(x)$.

1. $y'' - 4y' + 4y = 0$, $y_1 = e^{2x}$
2. $y'' + 16y = 0$, $y_1 = \cos(4x)$

Problem 2: Find a second solution by reduction of order - nonconstant coefficients

The given function $y_1(x)$ is a solution of the given differential equation. Use the reduction of order method to find a second solution $y_2(x)$.

1. $x^2y'' - 7xy' + 16y = 0$, $y_1 = x^4$
2. $xy'' + y' = 0$, $y_1 = \ln x$

Problem 3: Find a second solution by reduction of order - nonhomogeneous

The given function $y_1(x)$ is a solution of the associated homogeneous equation. Use the reduction of order method to find a solution $y(x) = u(x)y_1(x)$ of the nonhomogeneous equation.

1. $x^2y'' + xy' - 4y = x^3$, $y_1 = x^2$
2. $2x^2y'' + 3xy' - y = \dfrac{1}{x}$, $y_1 = x^{1/2}$

Problem 4: Reduction of order for higher order equation

Consider the homogeneous linear third order equation

$$xy''' - xy'' + y' - y = 0.$$

Given that $y_1(x) = e^x$ is a solution. Use the substitution $y = uy_1$ to reduce this third order equation to a homogeneous linear second order equation in the variable $w = u'$. You do not need to solve this second order equation.

Problem 5: Reduction of order for higher order equation

Consider the homogeneous linear third order equation

$$xy''' + (1 - x)y'' + xy' - y = 0.$$

Given that $y_1(x) = x$ is a solution. Use the substitution $y = uy_1$ to reduce this third order equation to a homogeneous linear second order equation in the variable $w = u'$. You do not need to solve this second order equation.